

# Introduction of Unsupervised Image Segmentation

Presenter:

Haw-Shiuan Chang(張浩軒)

Normalized cuts and image segmentation, PAMI 2000

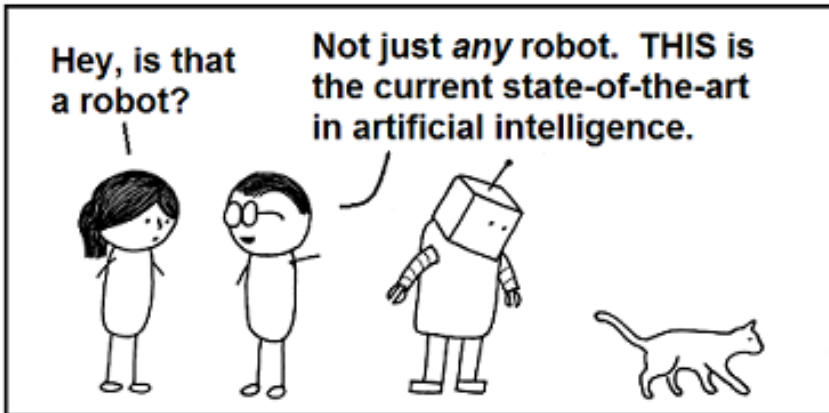
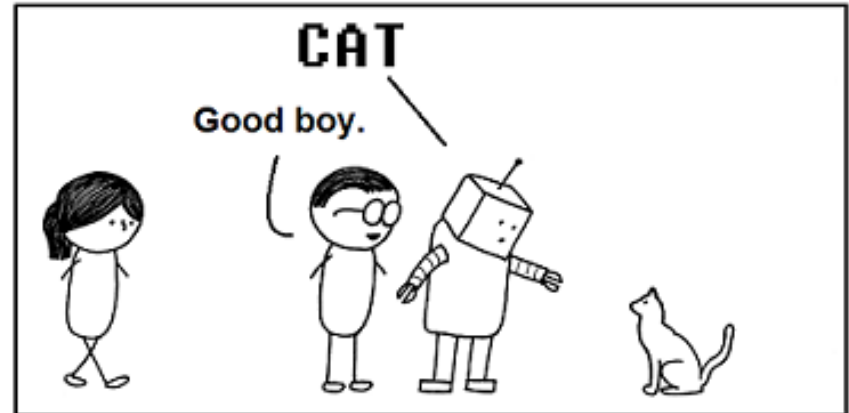
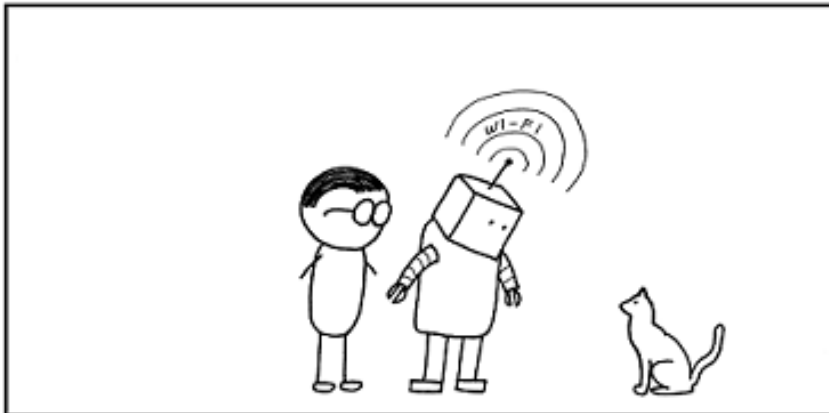
Mean shift: a robust approach toward feature space analysis, PAMI 2002

Efficient graph-based image segmentation, IJCV 2004

Contour Detection and Hierarchical Image Segmentation, PAMI 2011

Image Segmentation by Cascaded Region Agglomeration, CVPR 2013

# The Dream of Computer Vision



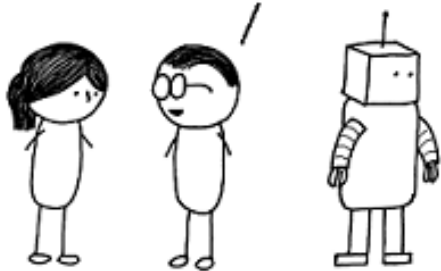
We trained a 9-layered locally connected sparse autoencoder with pooling and local contrast normalization on a dataset of 10 million images.



It was trained for 3 days on a cluster of 1000 machines comprising 16,000 cores.

# The Dream of Computer Vision

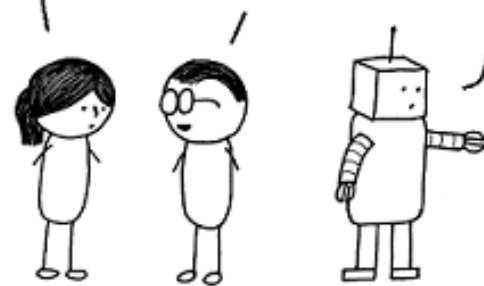
It can now recognize 22,000 different object categories with 15.8% accuracy.



15.8%?

Impressive, huh?

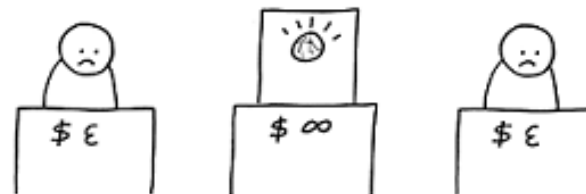
CAT



Impressive? Didn't you AI guys program a machine that defeated a world chess champion way back in 1997?

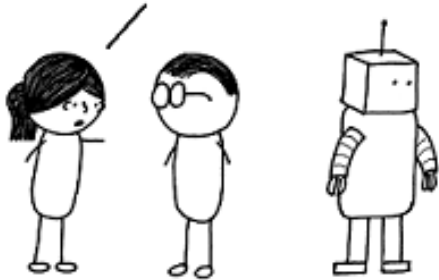


And didn't you AI guys build a computer system that kicked the respective asses of the two best *Jeopardy!* champions of all time?

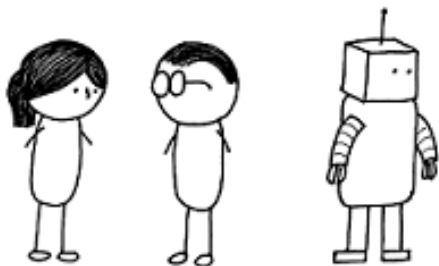
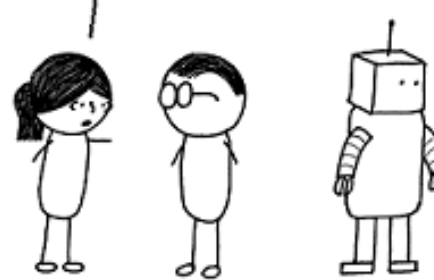


# The Dream of Computer Vision

What happened to 'HAL 9000 by 2001'?  
Where are our Series 800 Terminator robots?

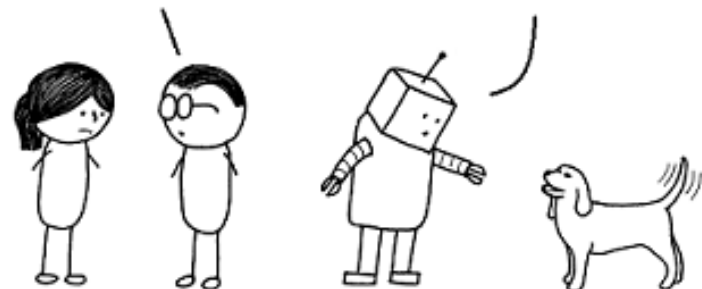


You call this  
progress?!!



Sorry we annoyed you  
with our awesomeness.

**CAT**



# Computer Vision

Registration

Score: Cat: 0.286; Dog: 0.3419



Score: Cat: 0.327; Dog: 0.188



Unsupervised segmentation

# Outline

- Introduction
- Methods
  - NCut
  - FH
  - Mean Shift
  - gPb-OWT-UCM
  - ISCRA
- Experimental Results
- Conclusion

# Introduction

**Supervised Image segmentation**

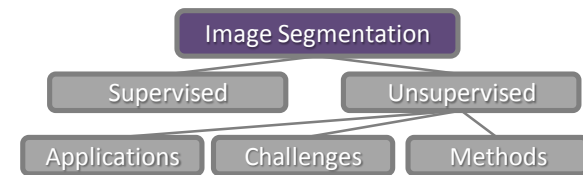
**Unsupervised Image segmentation**

**Application**

**Challenge**

**Methods**

# Image Segmentation



- *Image segmentation is one of the **fundamental** and **most studied** problems in computer vision.*

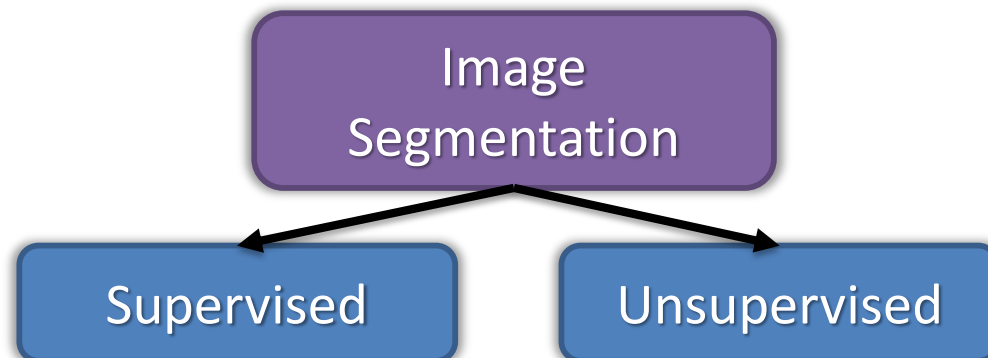
P. Arbelaez and L. Cohen

“Constrained Image Segmentation from Hierarchical Boundaries,” CVPR 2008

- *Image segmentation is a fundamental **low-level** vision problem with a **great potential** in applications.*

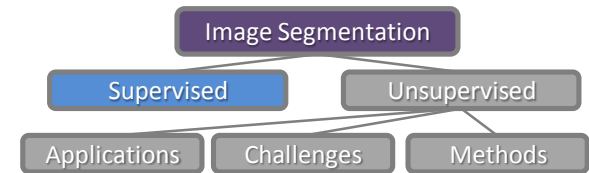
Z. Li, X. Wu, and S. Chang

“Segmentation using superpixels: a bipartite graph partitioning approach,” CVPR 2012





# Supervised vs. Unsupervised

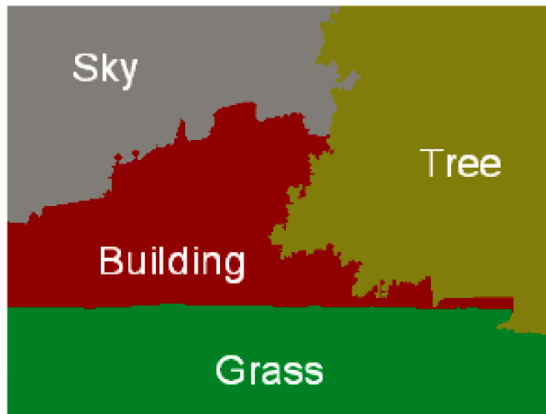


## ■ Supervised image segmentation (semantic segmentation)

Image



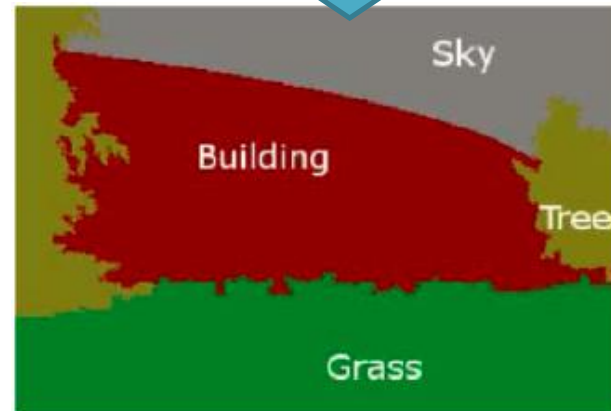
Labeled ground-truth



Training phase



Test Image

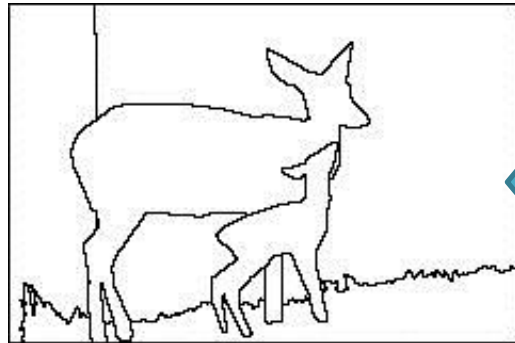
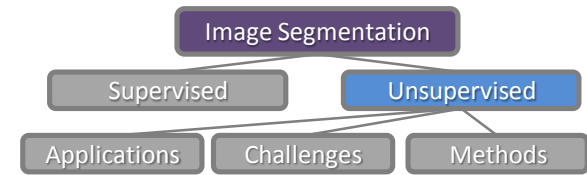


Goal

Testing phase

# Supervised vs. Unsupervised

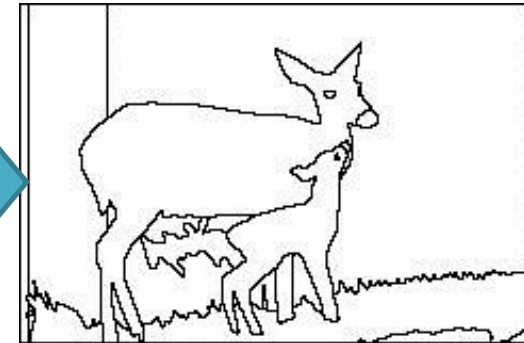
## ■ Unsupervised image segmentation



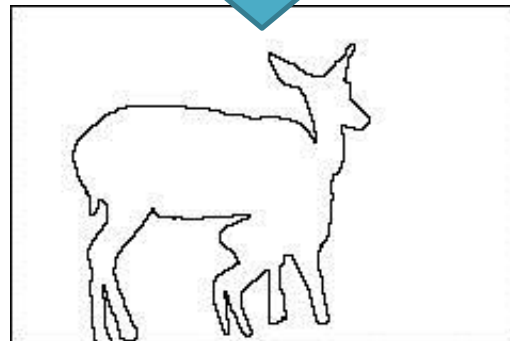
User # 1109



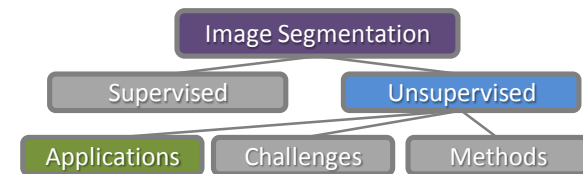
Image



User # 1124



User # 1116



# Applications

- Why segmentation?
  - To increase the accuracy of registration
  - Allow the algorithm with higher time complexity

- E.g., object recognition

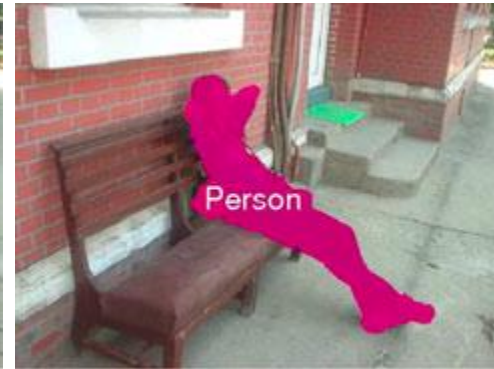
Object Recognition by Sequential Figure-Ground Ranking, IJCV 2012



Image



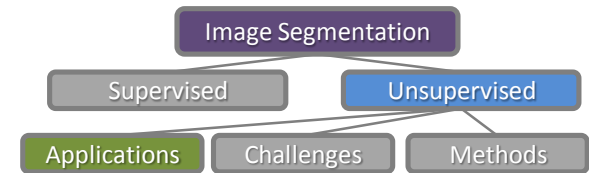
Traditional  
object recognition



Segmentation-based  
object recognition

If time complexity of recognition is  $O(p^2)$ , we often can reduce to  $O(sp^2)+O(p)$ , where  $p$  is pixel number,  $sp$  is superpixel number (e.g.  $p=150000$ ,  $sp=1000$ ).

# Applications



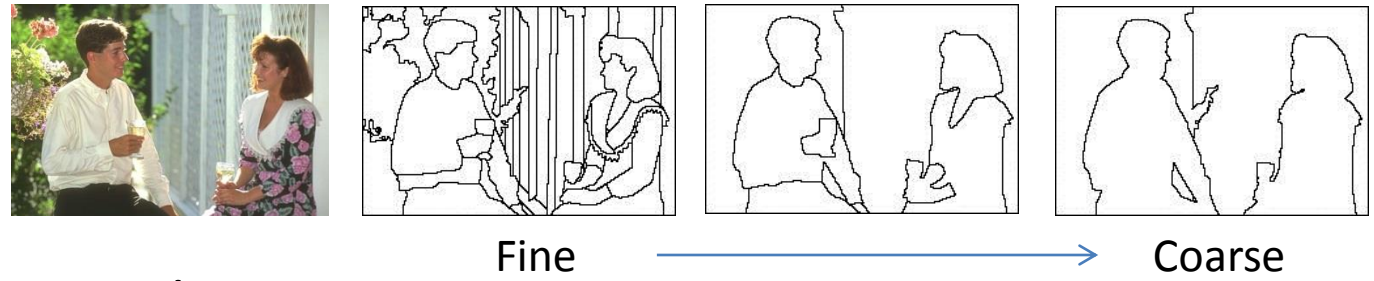
| Applications  | Main Components   |                                       |
|---|-------------------|---------------------------------------|
| Object recognition /<br>Semantic segmentation /<br>Saliency | Classification    | Unsupervised<br>Image<br>Segmentation |
| Image retrieval /<br>Object retrieval /<br>Co-segmentation  | Matching          |                                       |
| Video segmentation  | Motion estimation |                                       |
| Image editing /<br>Video editing                            | User interaction  |                                       |
| Video summary   | Some of the above |                                       |



# Challenges - objective function

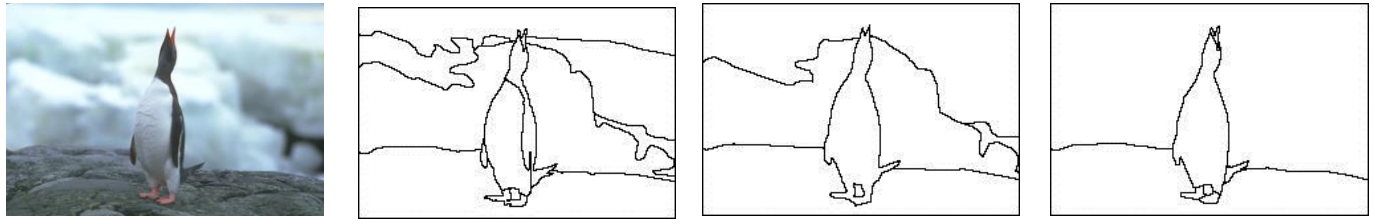
■ It is hard to define what good segmentation is because

■ Segmentation ambiguity

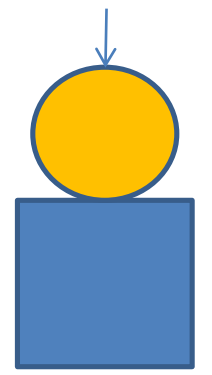


■ Semantic gap

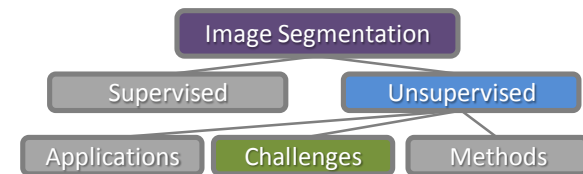
• feature{Color, texture, shape, etc.} ? → high-level concepts



Head or ball?  
Impossible to know!



■ Diverse image contexts (object types are unknown)

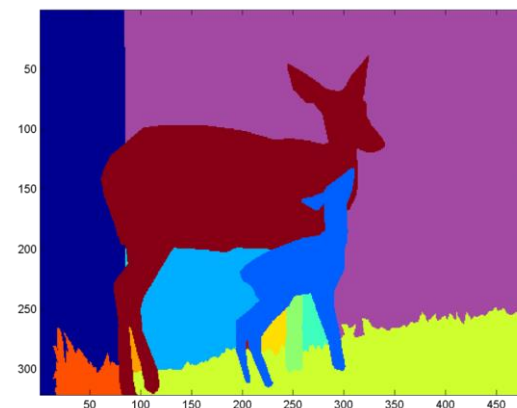


# Challenges - optimization

- Basically, it is a **NP-hard problem** for most of the objective functions.
  - Like most of unsupervised clustering tasks
- Enormous search space
  - The upper bound of search space size (# possible solution) is  $\frac{k^P}{k!}$  ( $\sim 13^{150000}$ )!
    - assuming that
      - the image is 300x500, # pixels (**P**) is 150000
      - ground truth have **k=13** segments
      - regardless connectivity constraint
- Enumerate the search space is difficult.
  - Because of connectivity constraint



input

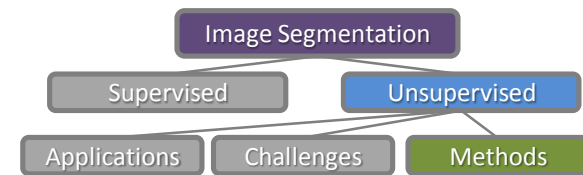


One output over  
 $\sim 13^{150000}$  possibility!

The number of pixels has been saw by human:

640 x 480 x 30 x 60 x 60 x 24 x 365 x 6,000,000,000 x 200,000  $\approx 10^{30}$   
 pixel          fps    secs   mins   hrs    days          people          years

# Methods



- I will introduce the methods which are **widely used** because of their
  - Simplicity
  - Efficiency
  - Good performance

|                                |                                | Performance                        | Efficiency                   | Complexity<br>(# parameters) | Theoretical support |
|--------------------------------|--------------------------------|------------------------------------|------------------------------|------------------------------|---------------------|
| NCut                           | (2000)                         | Bad<br>(original version)          | Reasonable                   | Simple                       | Best                |
| FH                             | (2004)                         | Reasonable<br>(↑ for superpixels)  | Fastest                      | Simple                       | Empirical           |
| Mean shift                     | (2002)                         | Reasonable<br>(↑ for oversegments) | Reasonable                   | Simple                       | Good                |
| gPb-OWT-UCM<br>(need training) | (2011)                         | Good                               | Slow                         | Complex                      | Empirical           |
| ISCRA                          | (2013+)<br>(code not released) | Best<br>(state of the art)         | Slowest<br>(need to use gPb) | Complex                      | Empirical           |

# Methods

**Ncut (Normalized Cut)**

FH (Efficient graph-based image segmentation)

Mean Shift

gPb-OWT-UCM

ISCRA (Image Segmentation by Cascaded Region Agglomeration)

Citation: 7931  
(since year 2000)





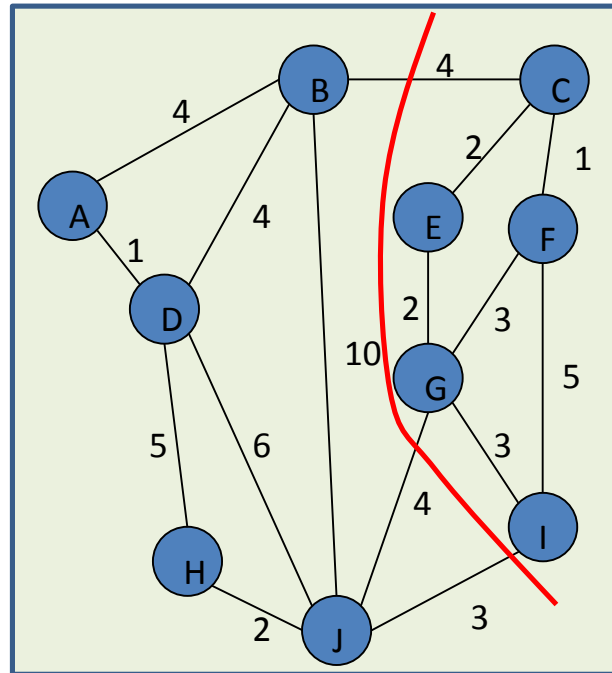
# Normalized Cut

Normalized cuts and image segmentation, PAMI 2000

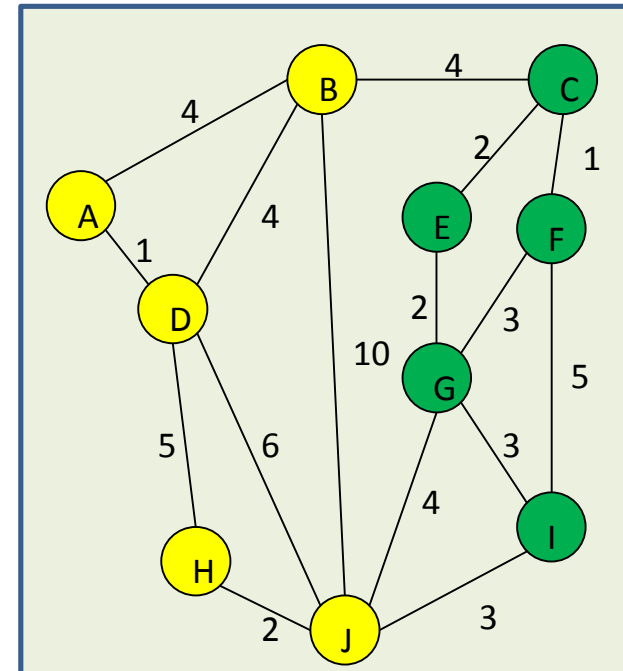
- NCut is one type of **graph based clustering** techniques.
  - similar to the spectral clustering
- NCut often becomes a component of other segmentation algorithms.



Assuming that we want to divide the image into **2** segments

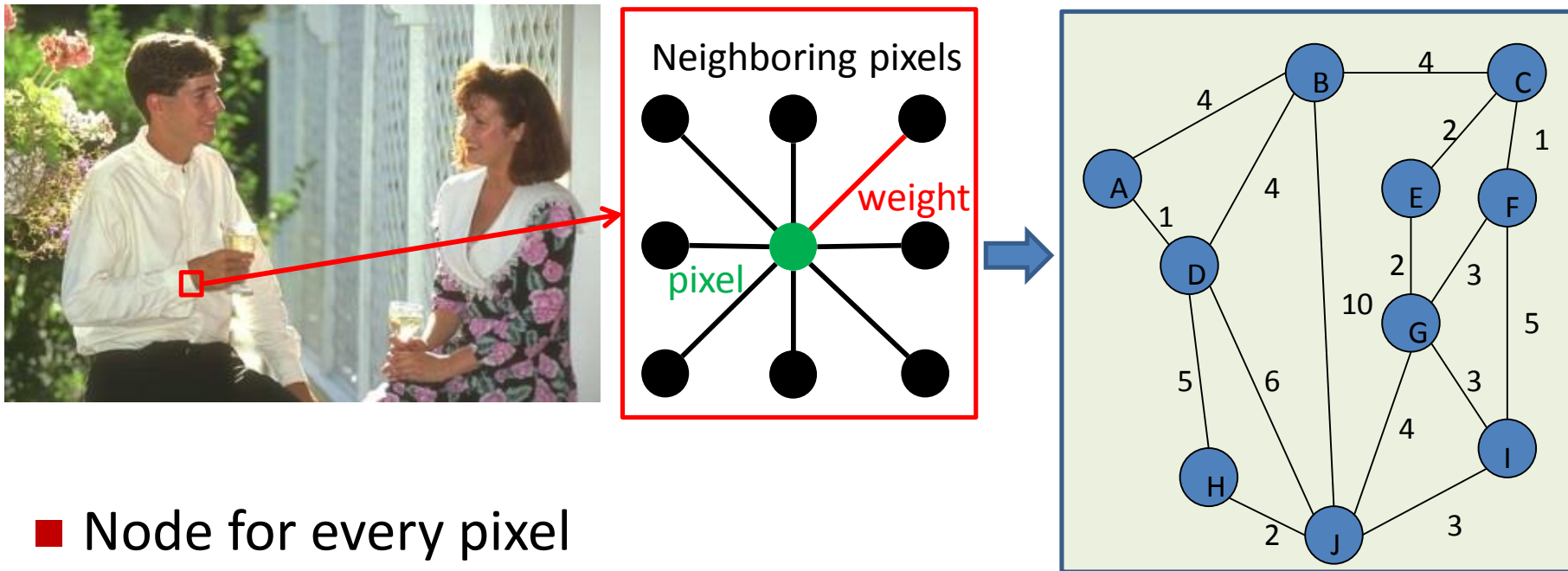


Graph



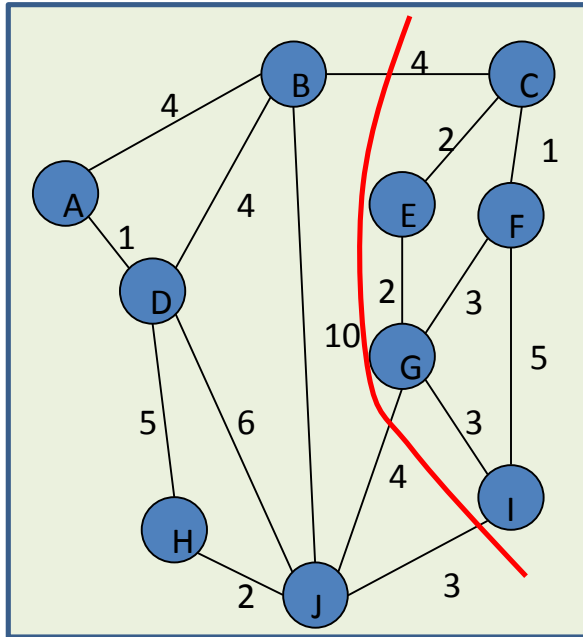
Clustered result

# Images as Graphs

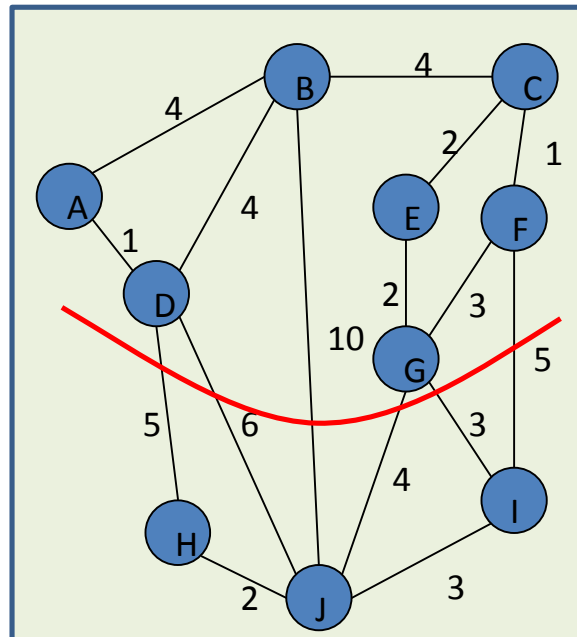


- Node for every pixel
- Edges between “neighbors”
- Edge weights/capacities are some measure of **similarity**.

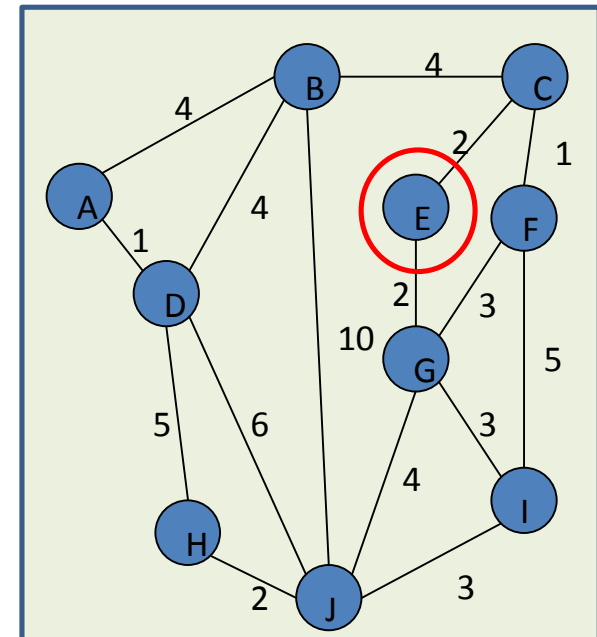
# What is a good segmentation?



(a)



(b)



(c)

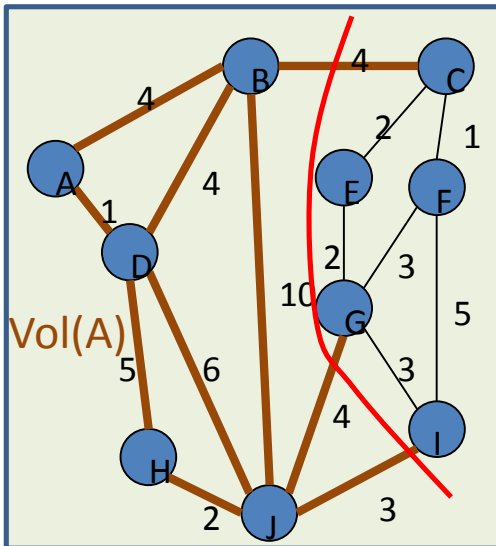
- (a) is better than (b) because it cuts edges with smaller weights.
- (a) is better than (c) because the clusters having similar sizes.

# Objective function

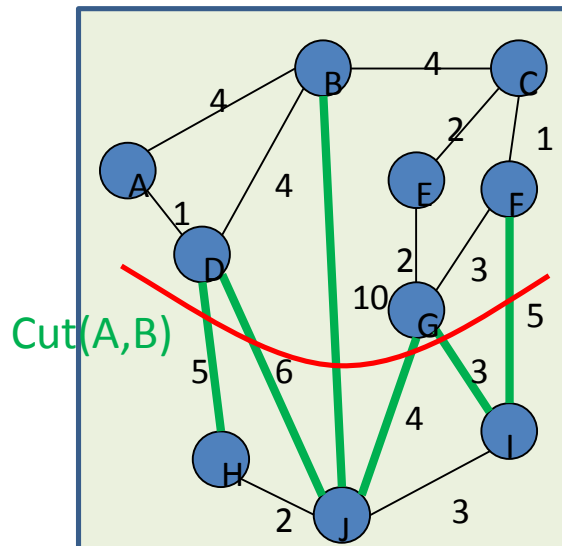
The weights between segment A and B

$$\text{minimize } \rightarrow NCut(A, B) = Cut(A, B) \left( \frac{1}{vol(A)} + \frac{1}{vol(B)} \right)$$

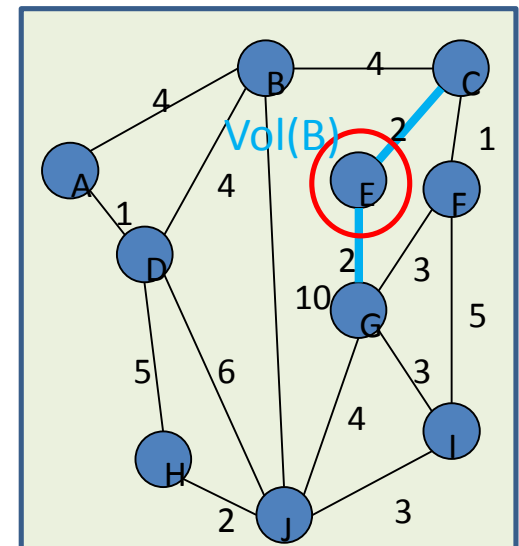
The weights within segment A



$NCut=0.66 \rightarrow \text{best}$   
 $(11/43+11/27)$



$NCut=1.48$   
 $(33/54+33/38)$

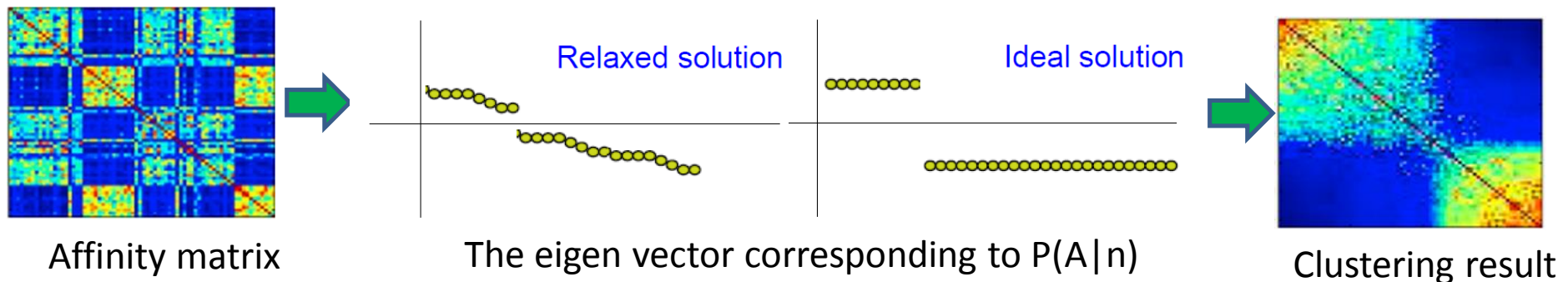


$NCut=1.07$   
 $(4/59+4/4)$

# Optimization

$$NCut(A, B) = Cut(A, B) \left( \frac{1}{vol(A)} + \frac{1}{vol(B)} \right)$$

- The problem is **NP-hard**.
  - Finding the global minimum needs to exam all possible cuts.
- Relax the problem (**discrete** → **continual**)
  - The pixel (node  $n$ ) belonging to segments  $P(A|n)$ 
    - e.g.,  $P(A|n)=0.6$  and  $P(B|n)=0.4$
- After some derivations, NCut turns out to be an **eigenvalue problem**.

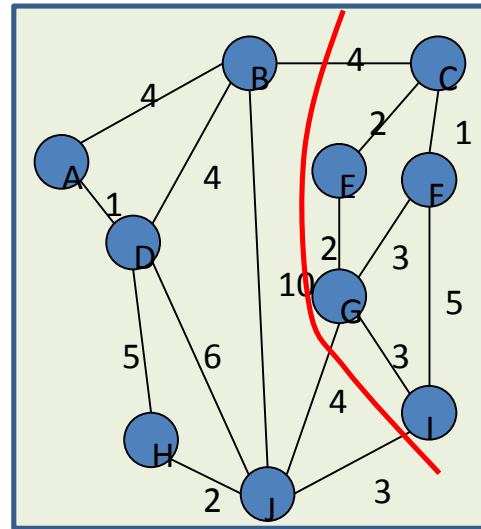


# Conclusion - NCut

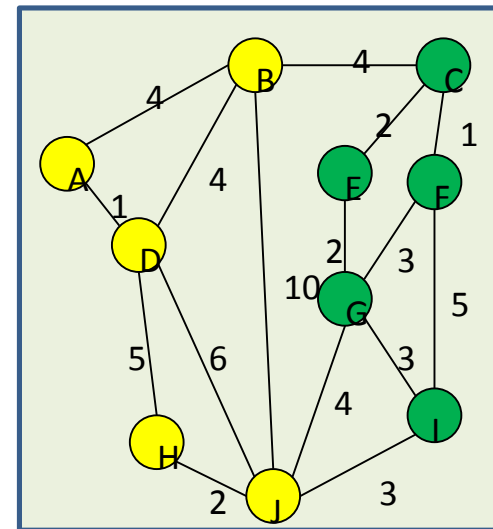
Normalized cuts and image segmentation, PAMI 2000



We repeat the process until the ideal # segments is reached.



Graph



Clustered results

|             | Performance               | Efficiency | Complexity<br>(# parameters) | Theoretical support |
|-------------|---------------------------|------------|------------------------------|---------------------|
| Ncut (2000) | Bad<br>(original version) | Reasonable | Simple                       | Best                |

The main limitation is the assumption that the segments tend to have same sizes, but the drawback could be alleviated if we use more sophisticated methods to build the graph.

# Methods

Ncut (Normalized Cut)

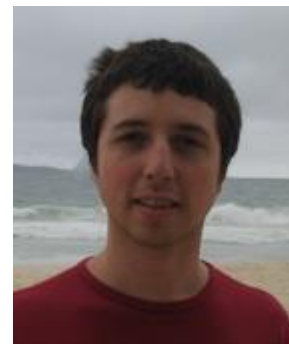
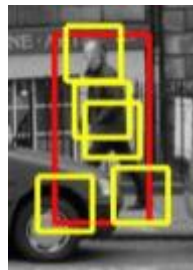
**FH (Efficient graph-based image segmentation)**

Mean Shift

gPb-OWT-UCM

ISCRA (Image Segmentation by Cascaded Region Agglomeration)

Citation: 2133  
(since year 2004)



Pedro  
Felzenszwalb



Daniel  
Huttenlocher

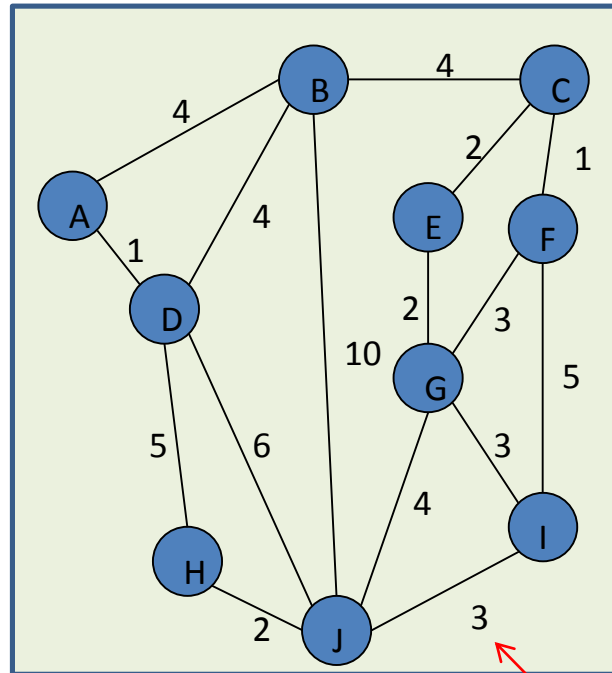
# Efficient graph-based image segmentation

Efficient graph-based image segmentation, IJCV 2004

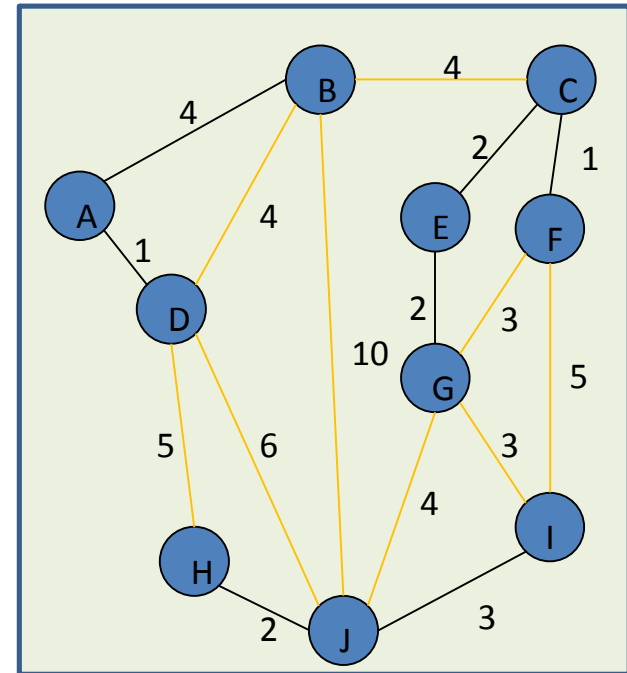
- Similar to **single linkage clustering** or minimal spanning tree
- Widely used in video segmentation



Image



Graph

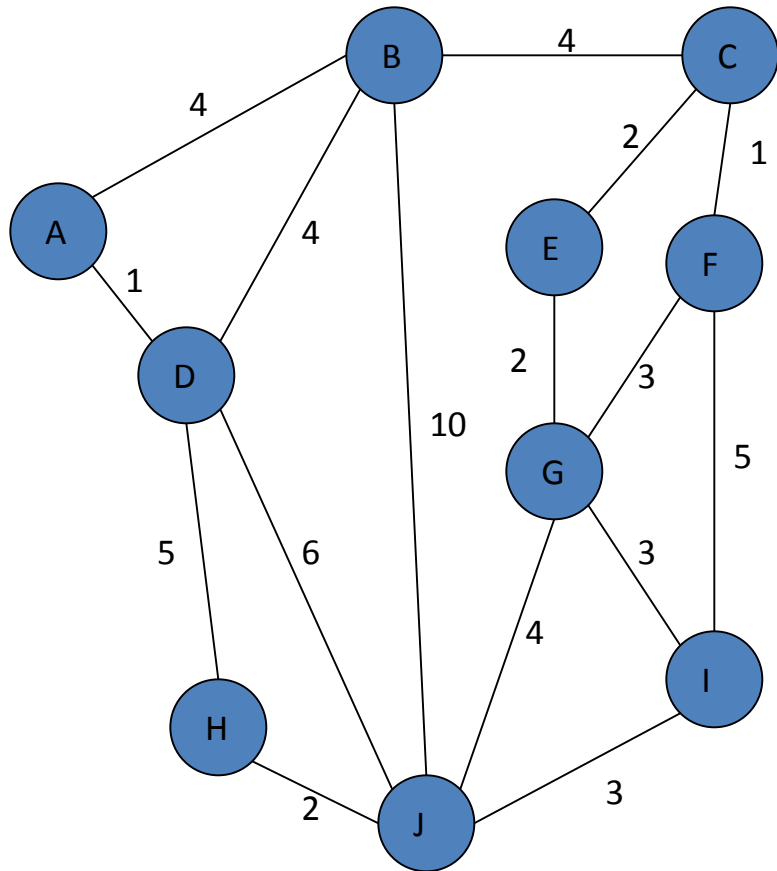


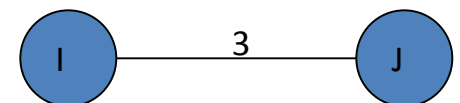
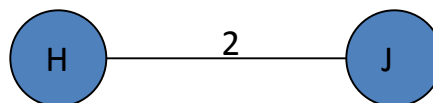
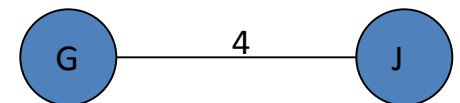
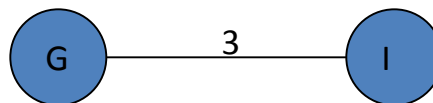
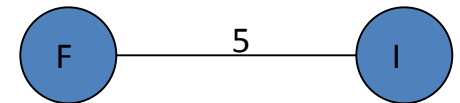
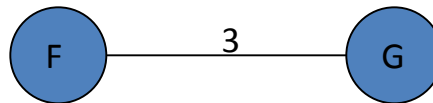
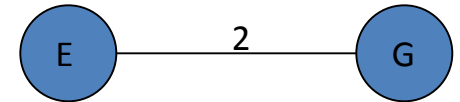
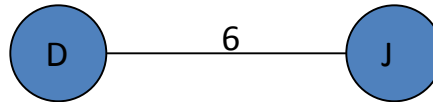
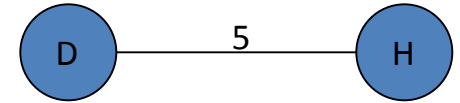
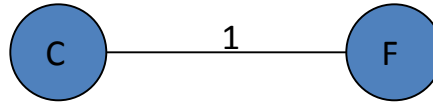
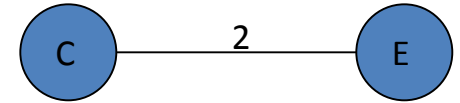
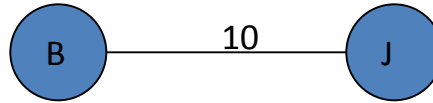
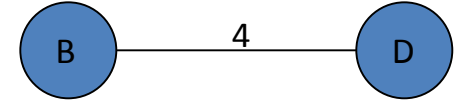
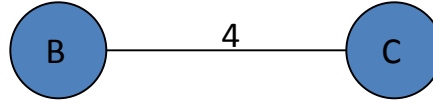
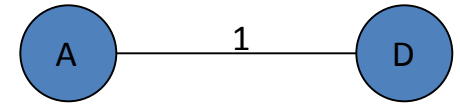
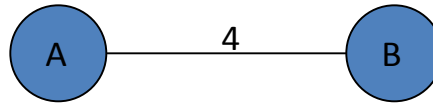
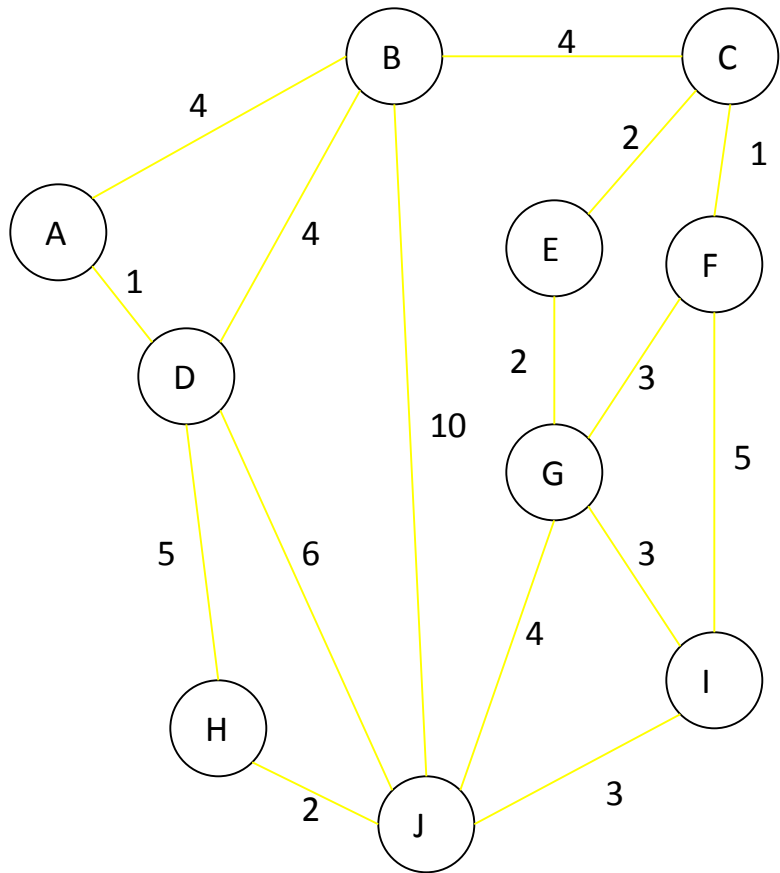
Clustered results

dissimilarity

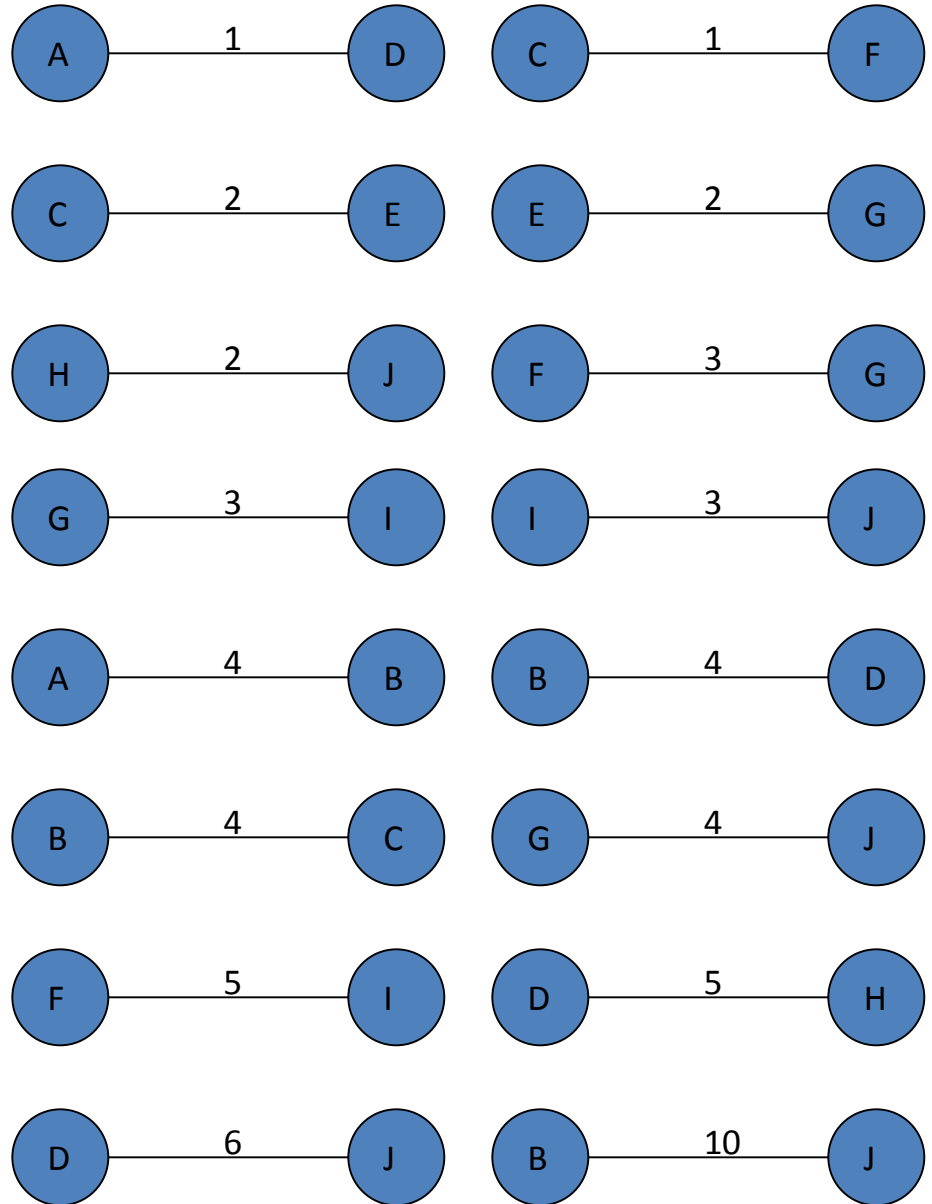
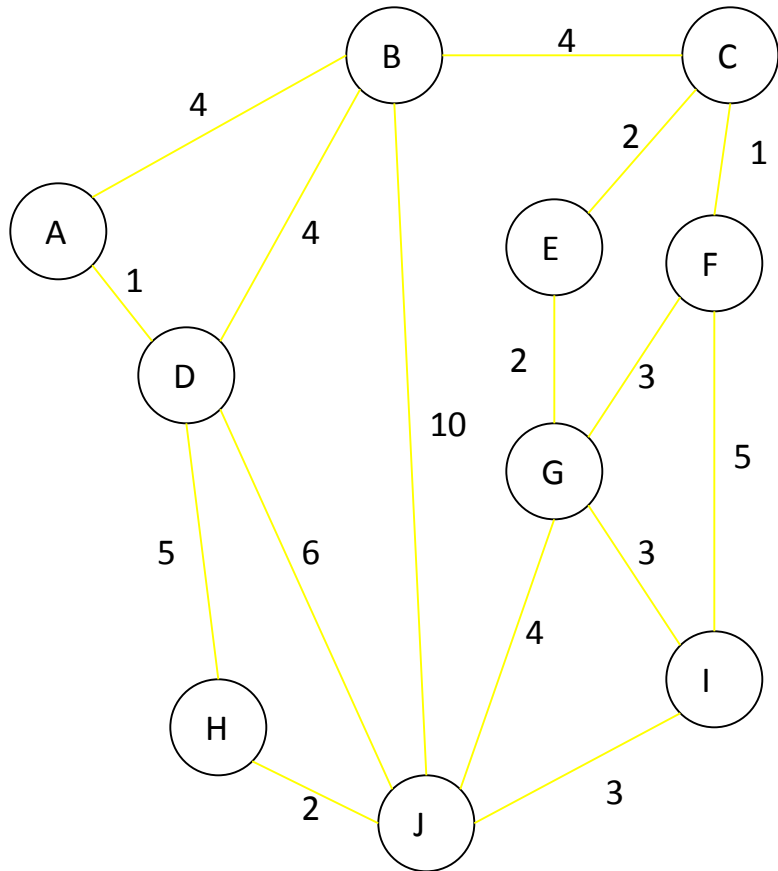


# Image Graph

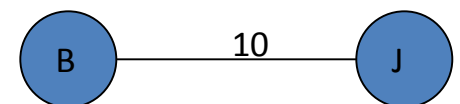
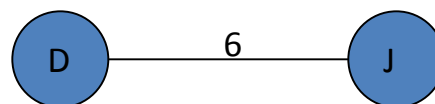
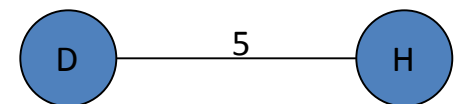
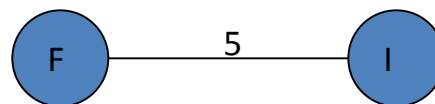
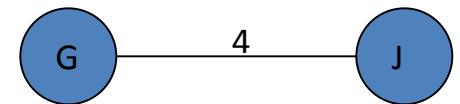
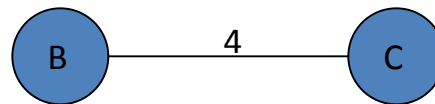
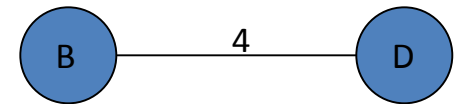
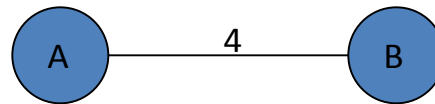
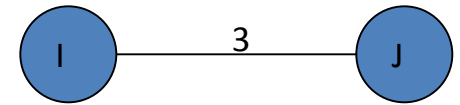
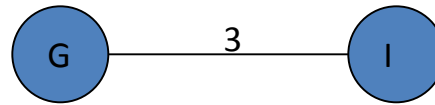
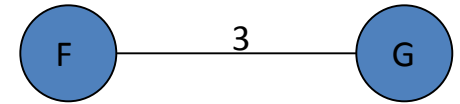
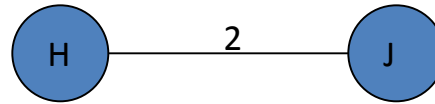
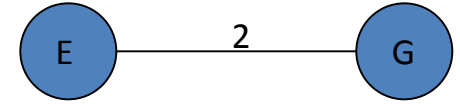
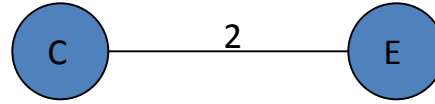
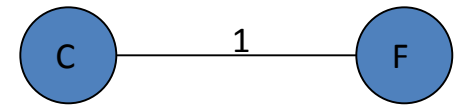
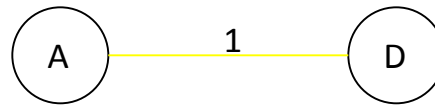
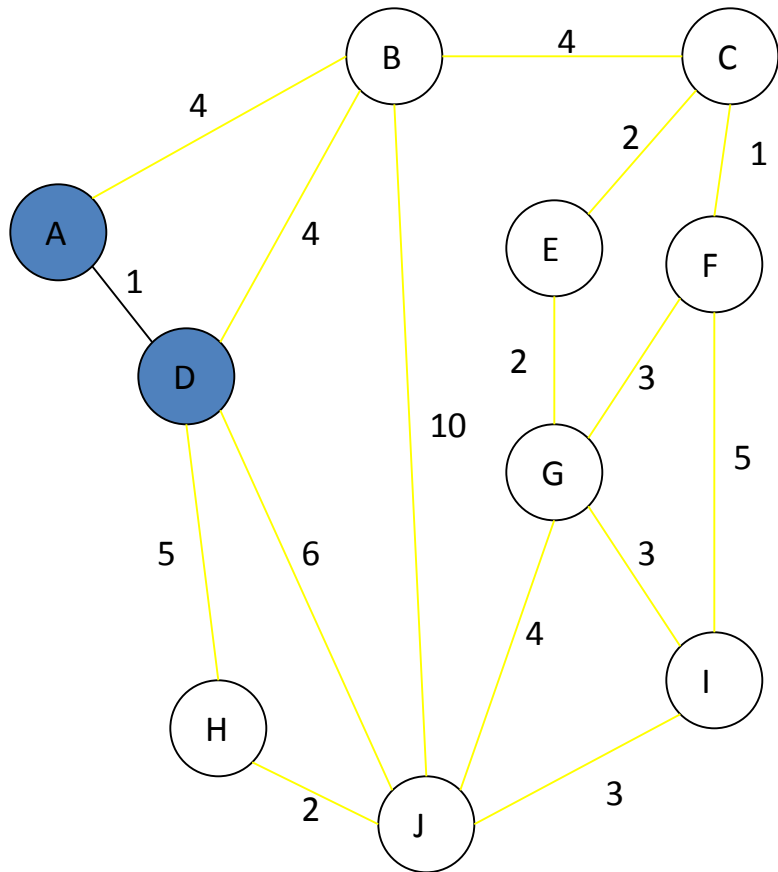




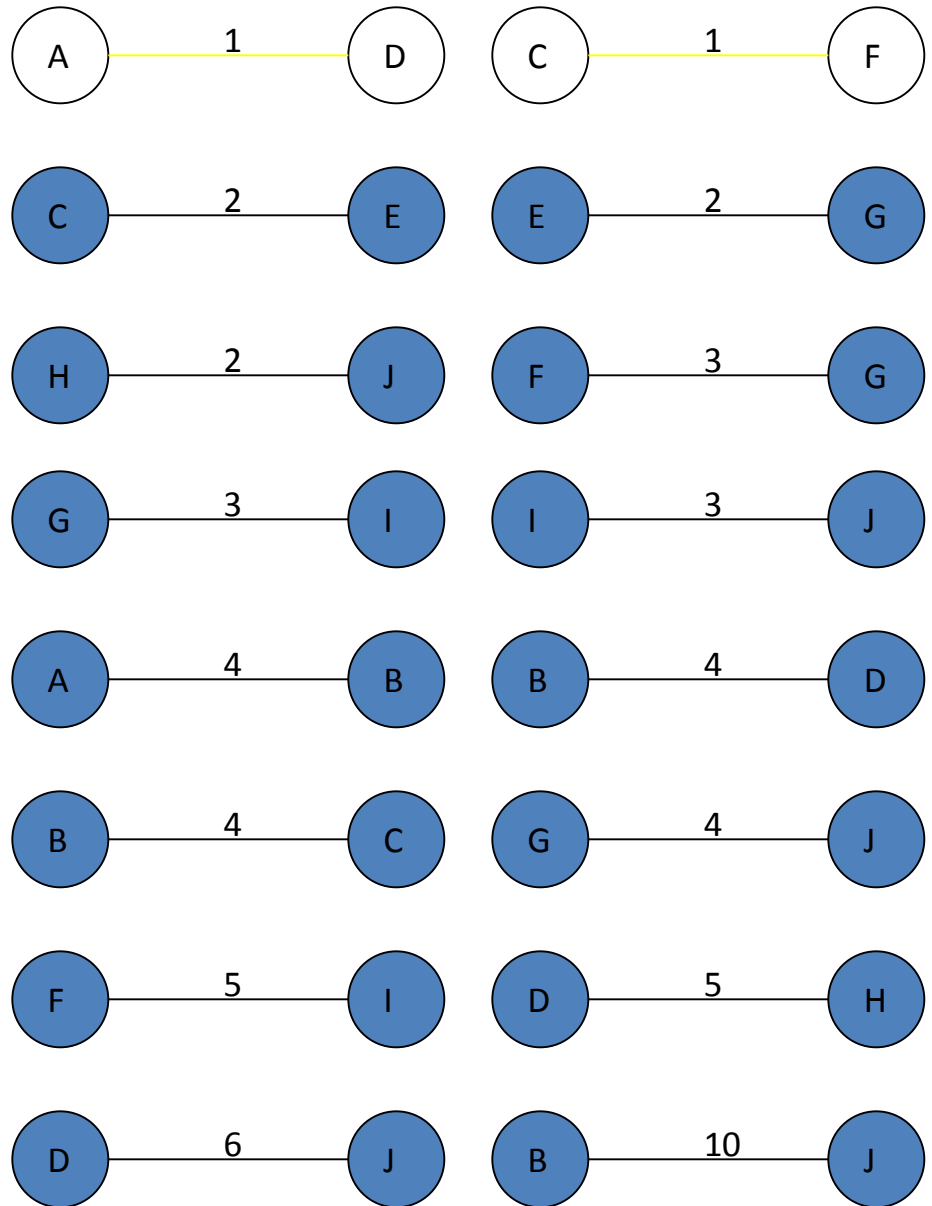
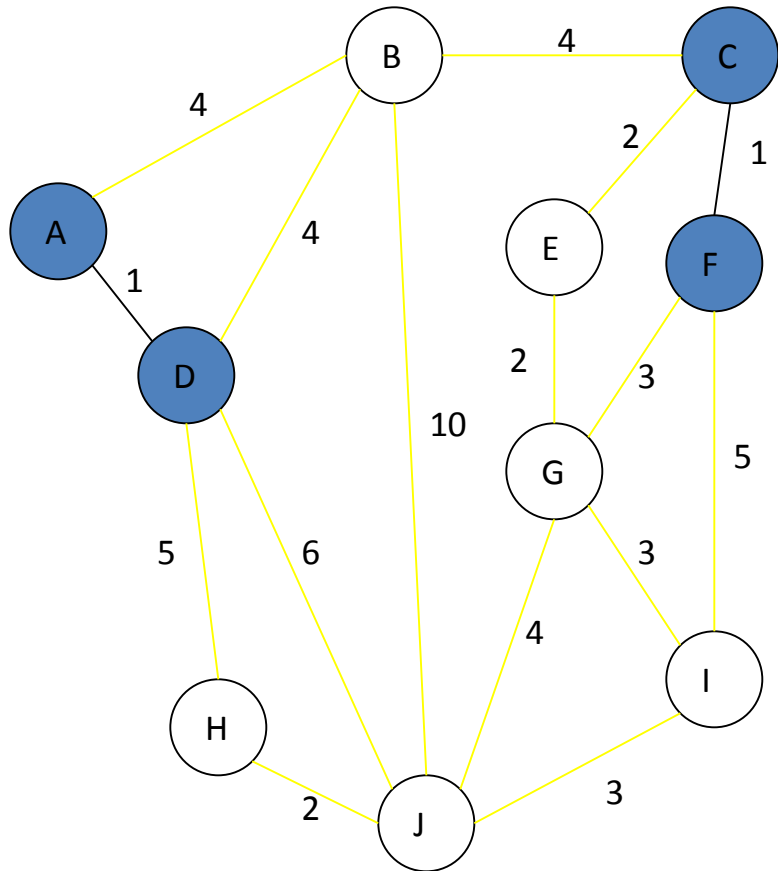
# Sort Edges



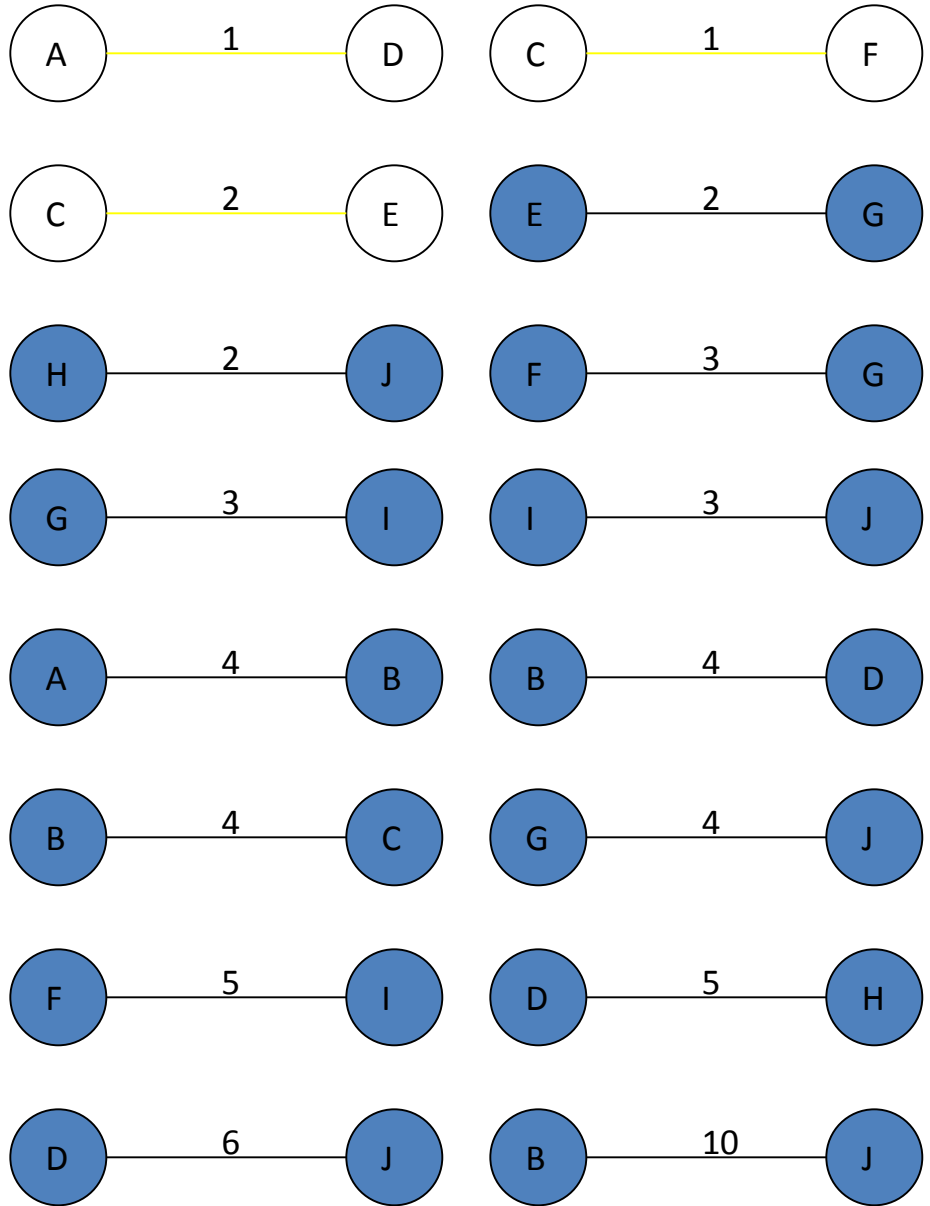
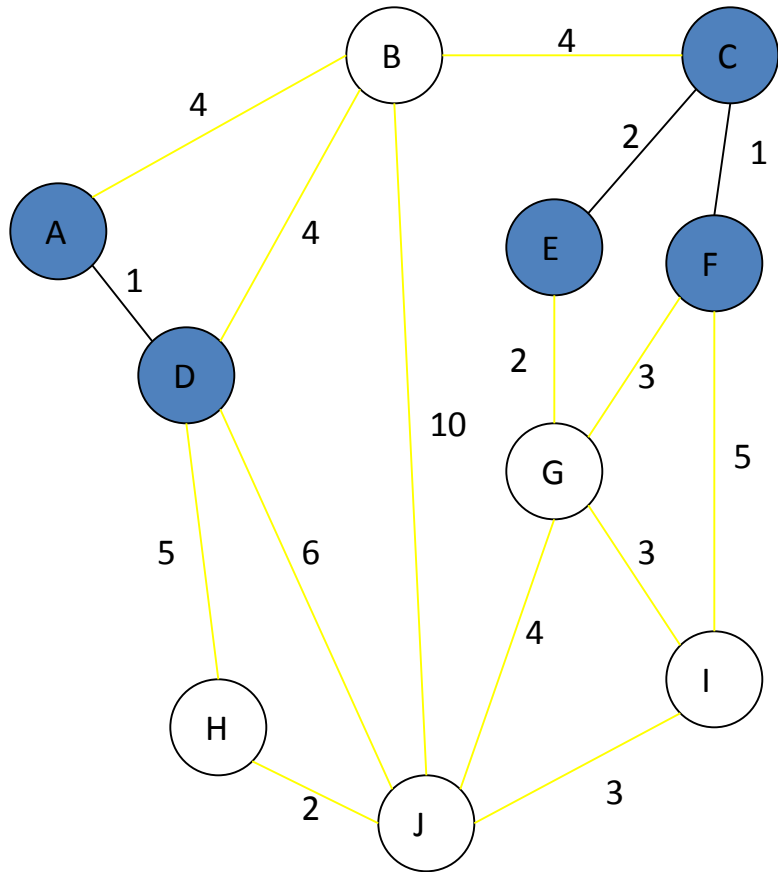
Add Edge



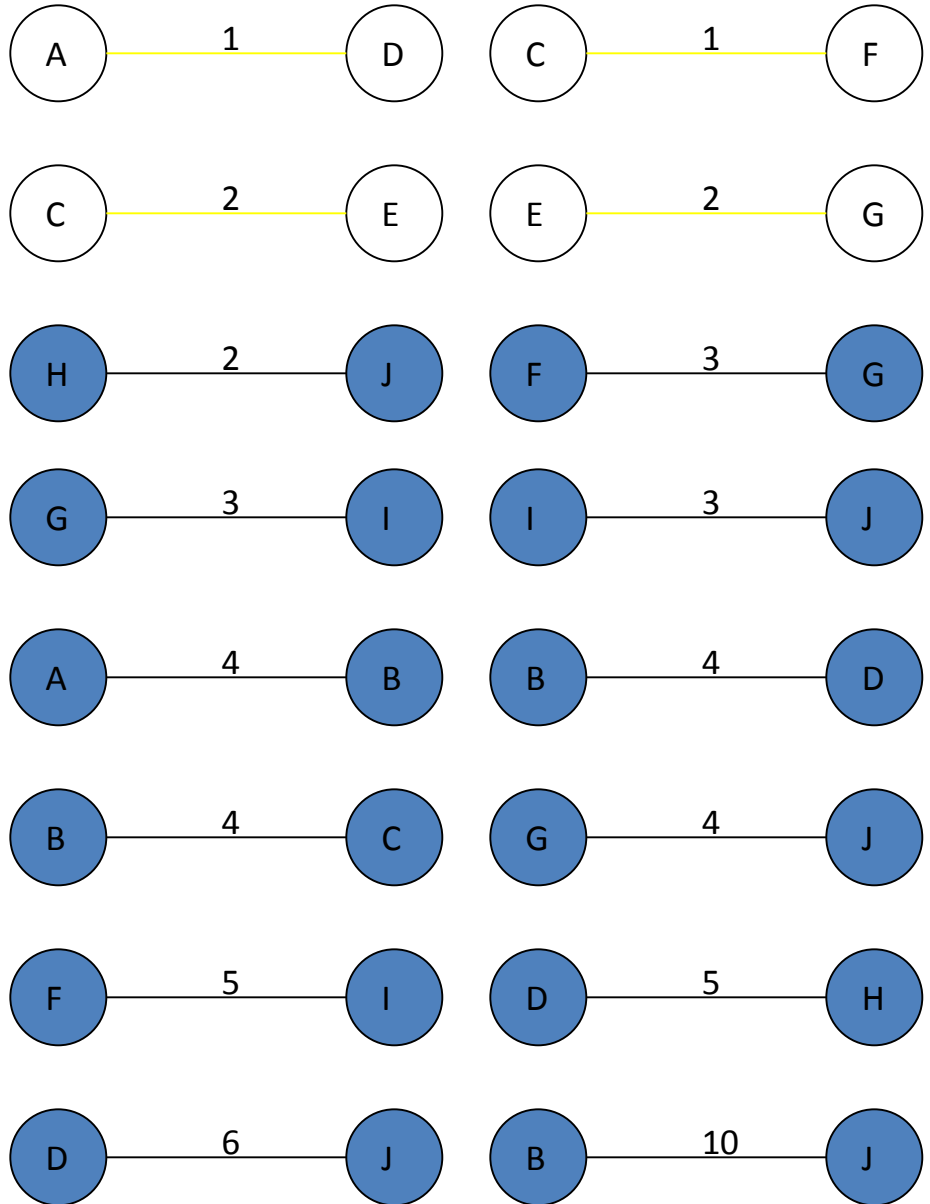
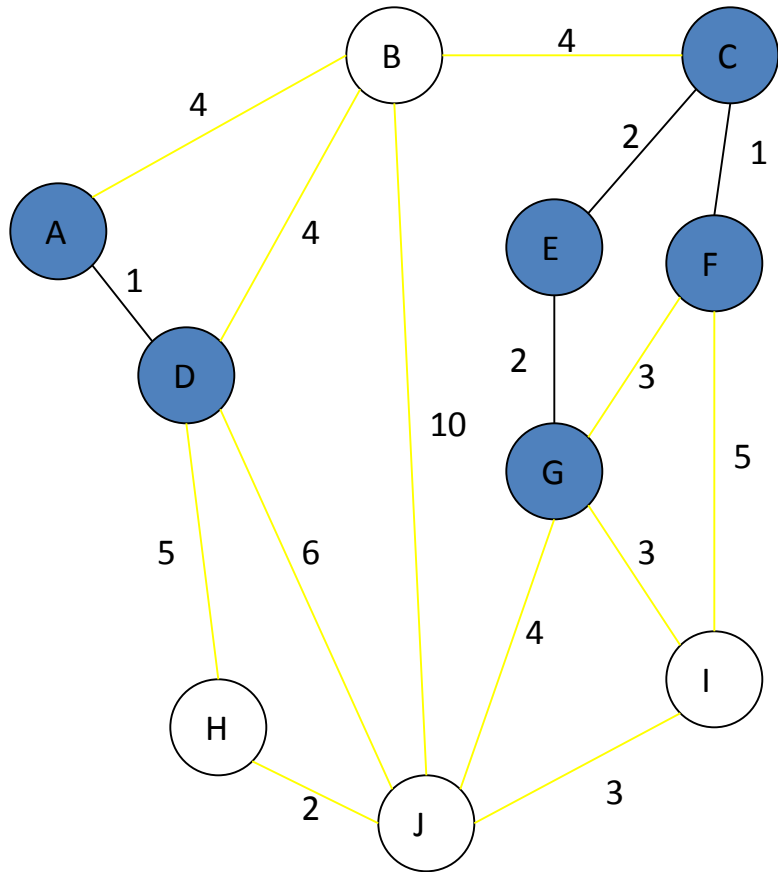
Add Edge



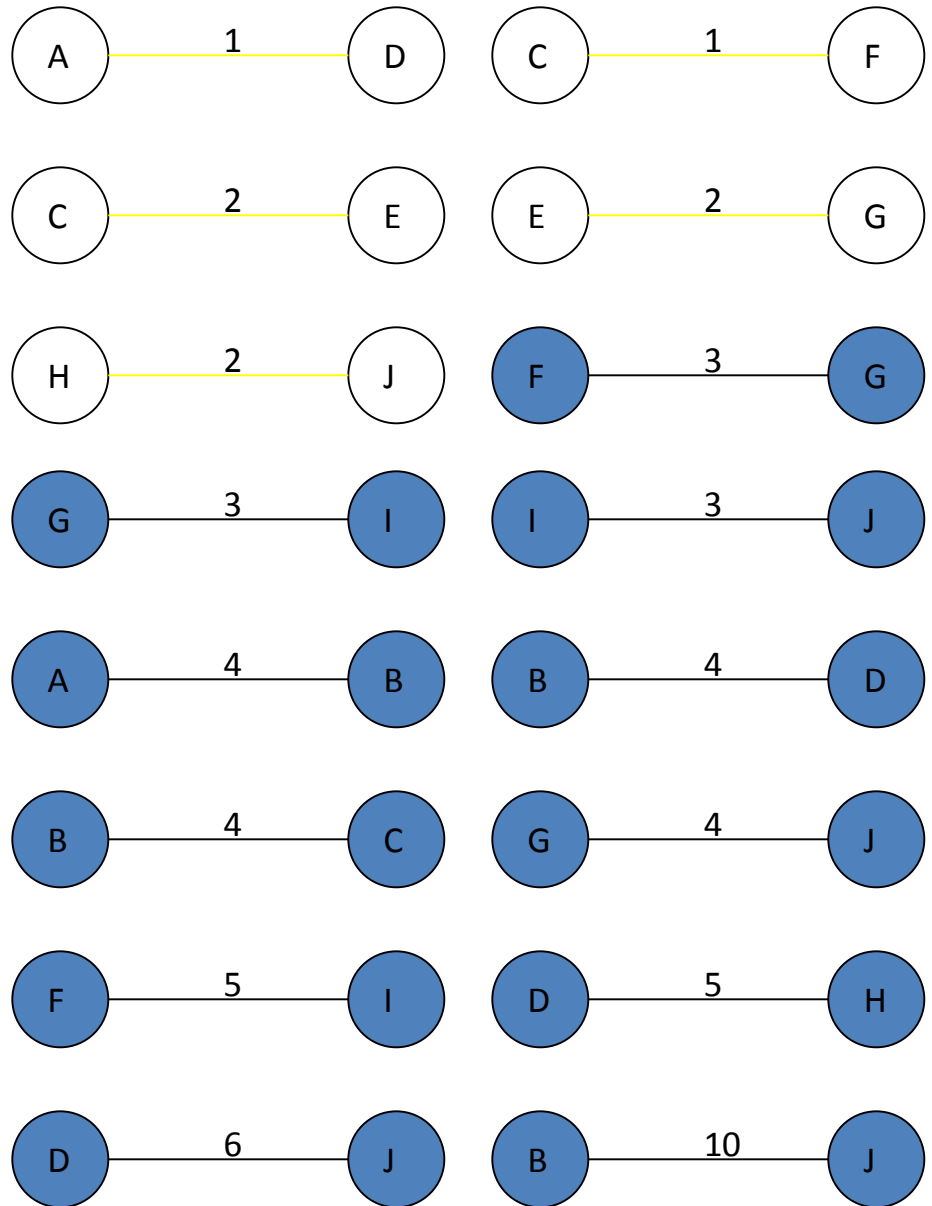
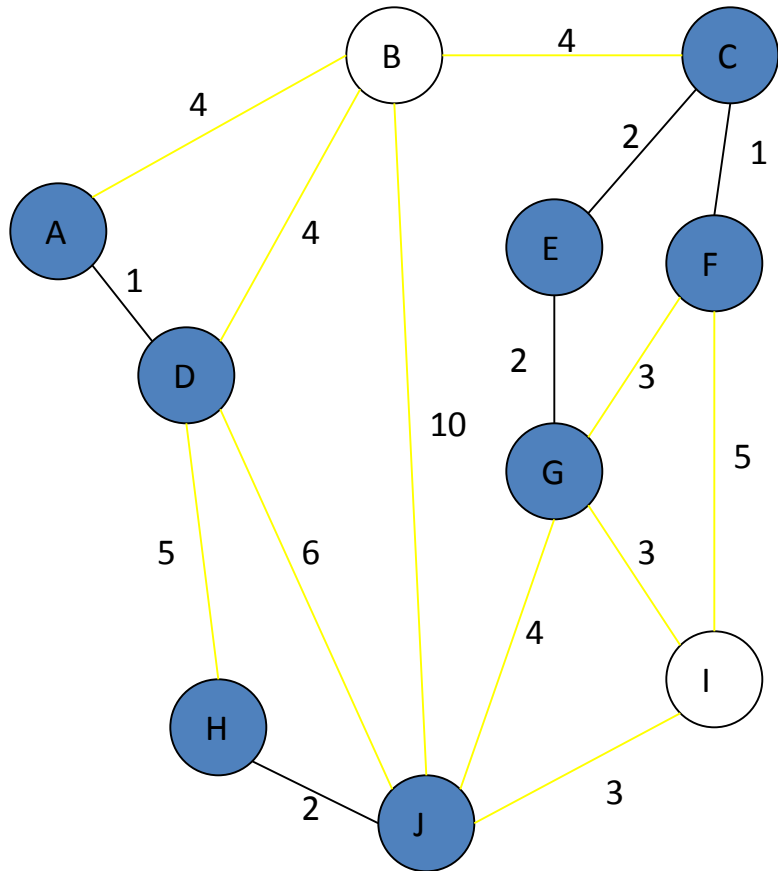
Add Edge



Add Edge



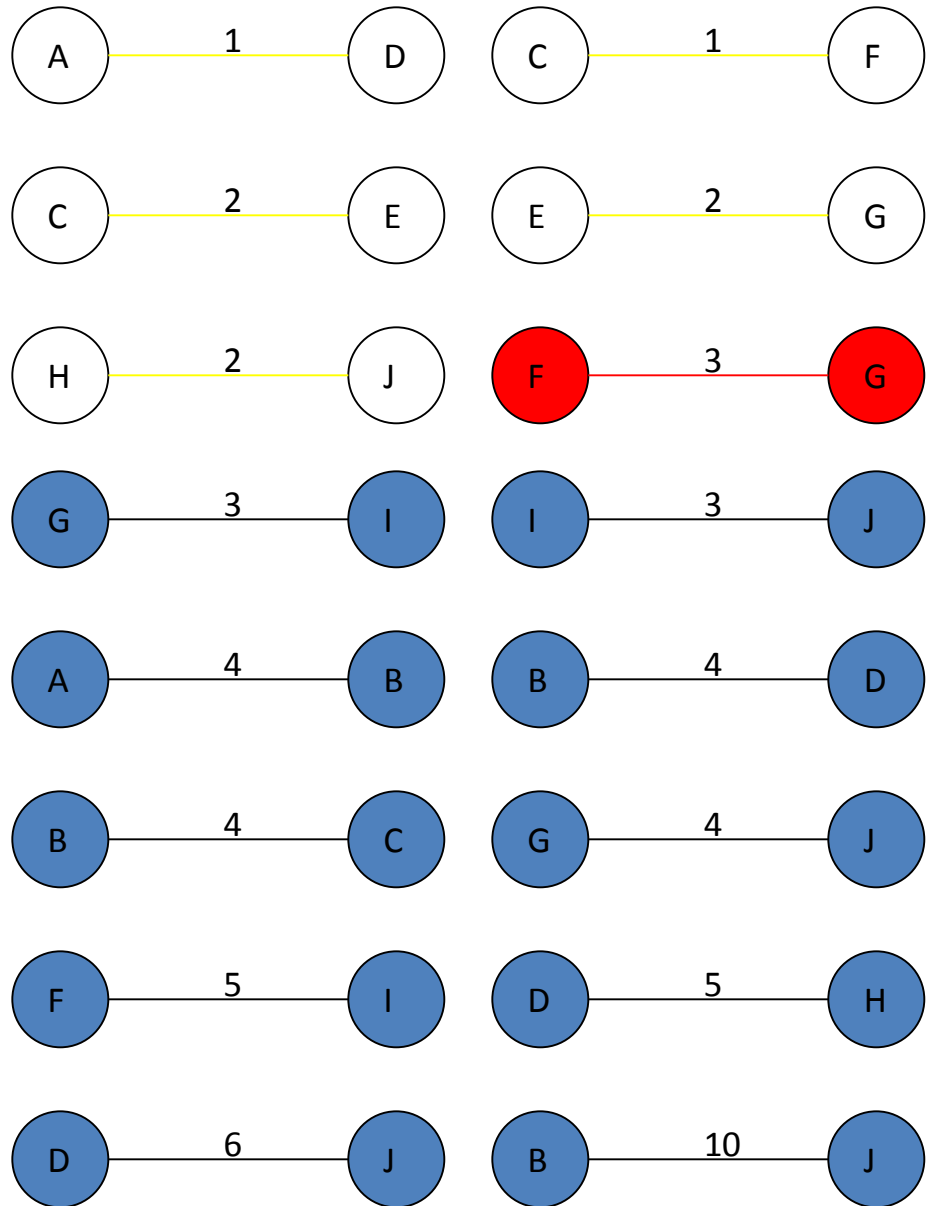
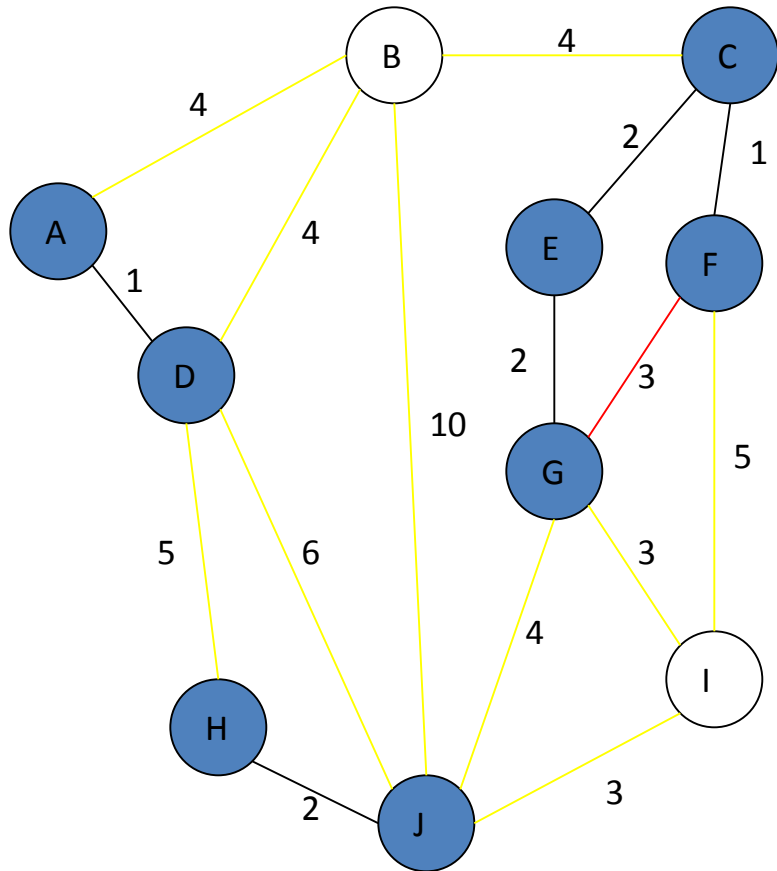
Add Edge





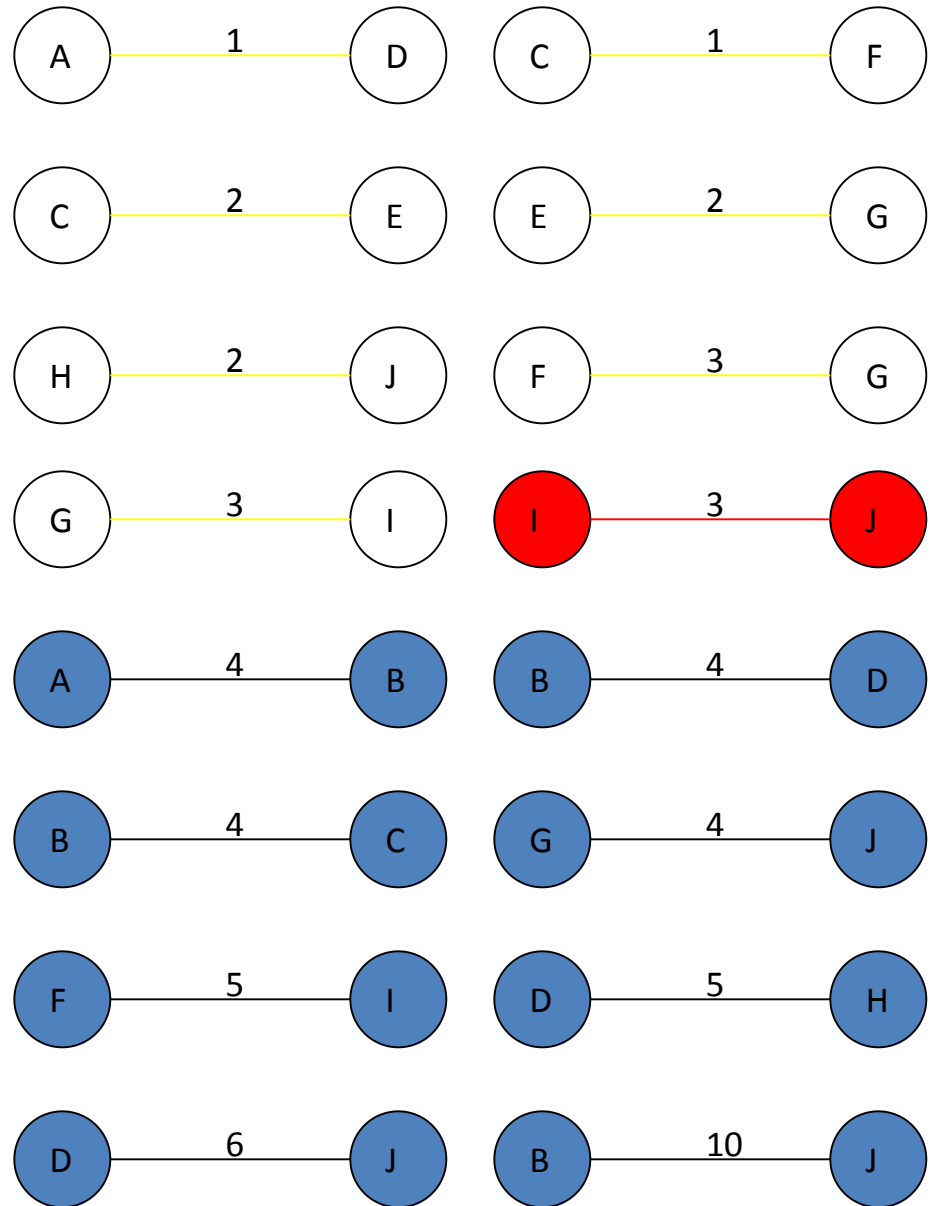
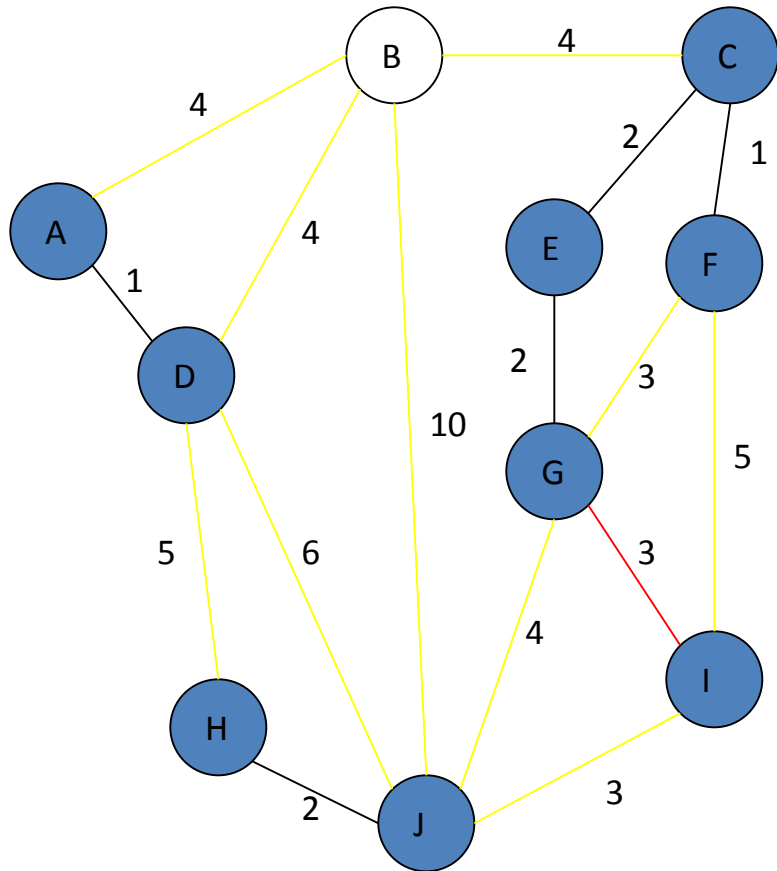
Cycle

Don't Add Edge

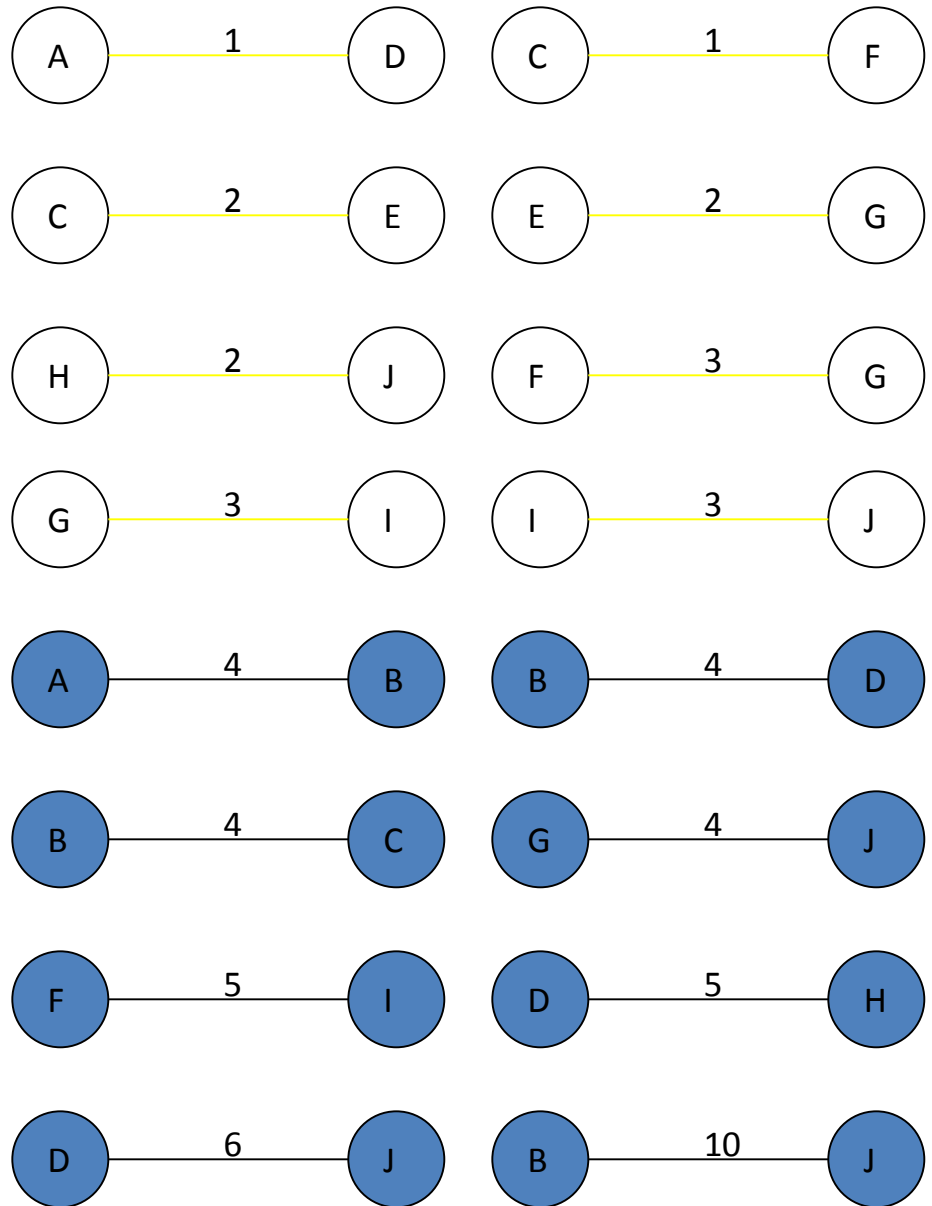
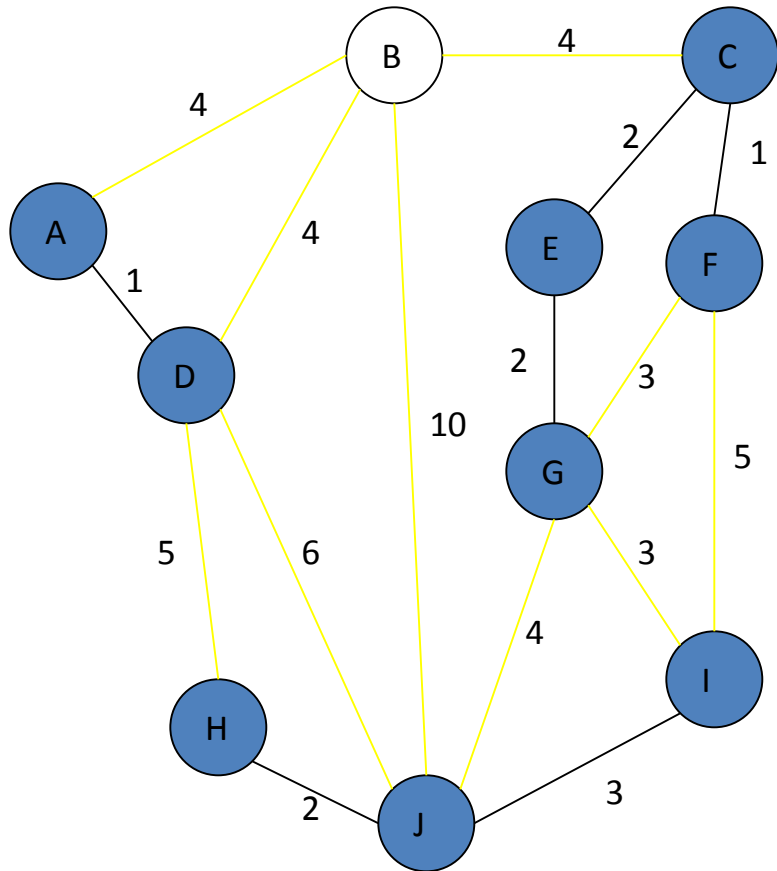


Higher than a dynamic threshold!

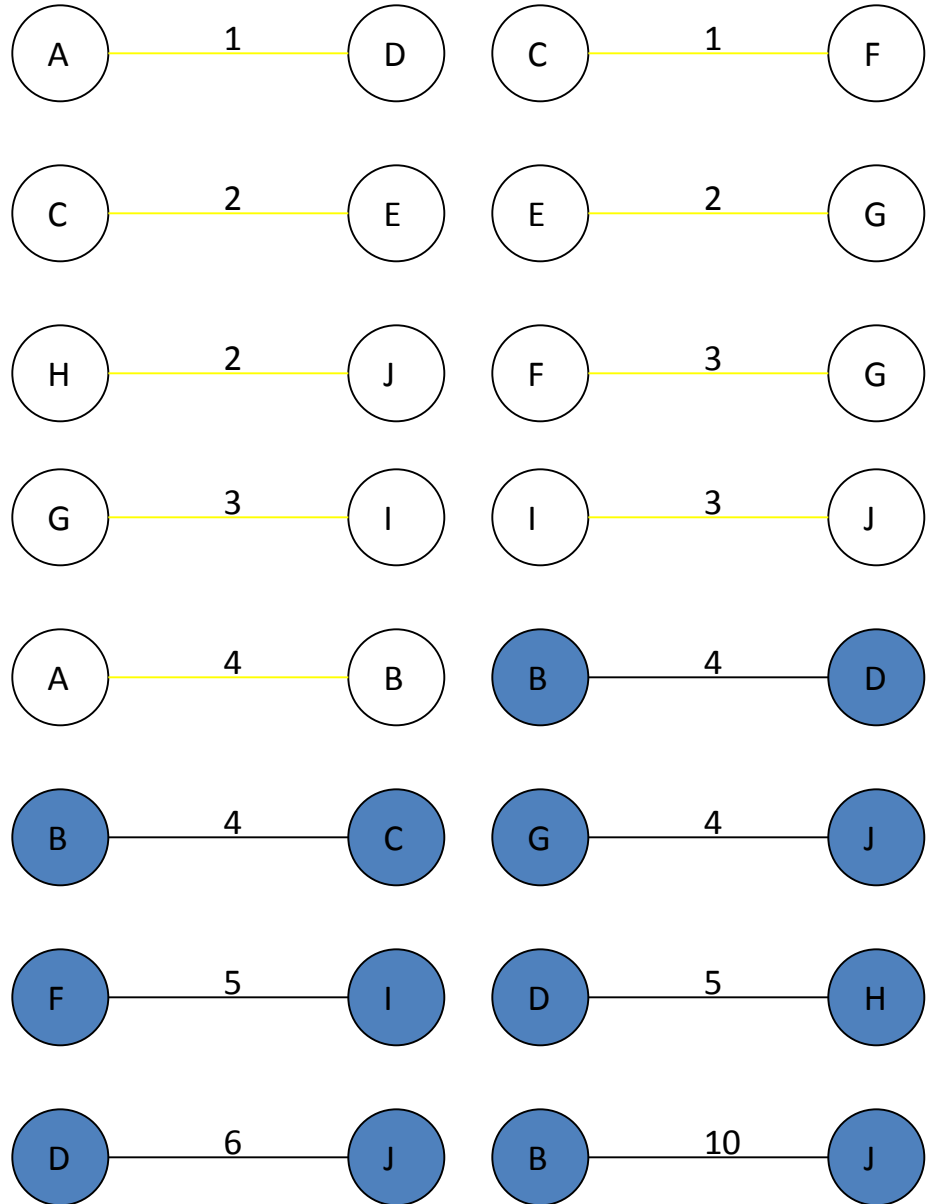
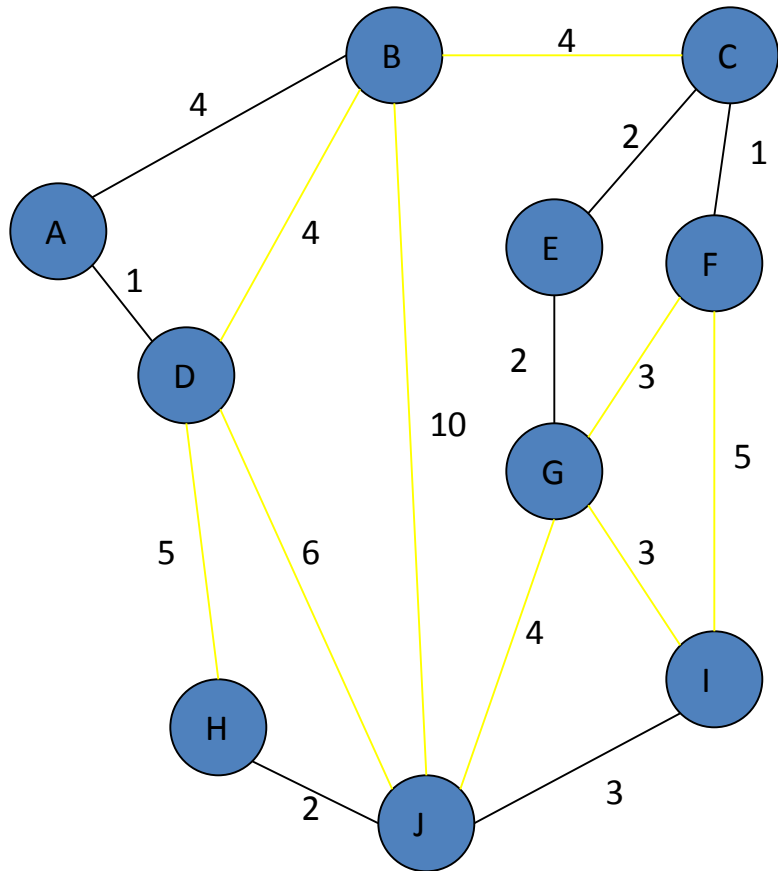
Don't Add Edge



Add Edge

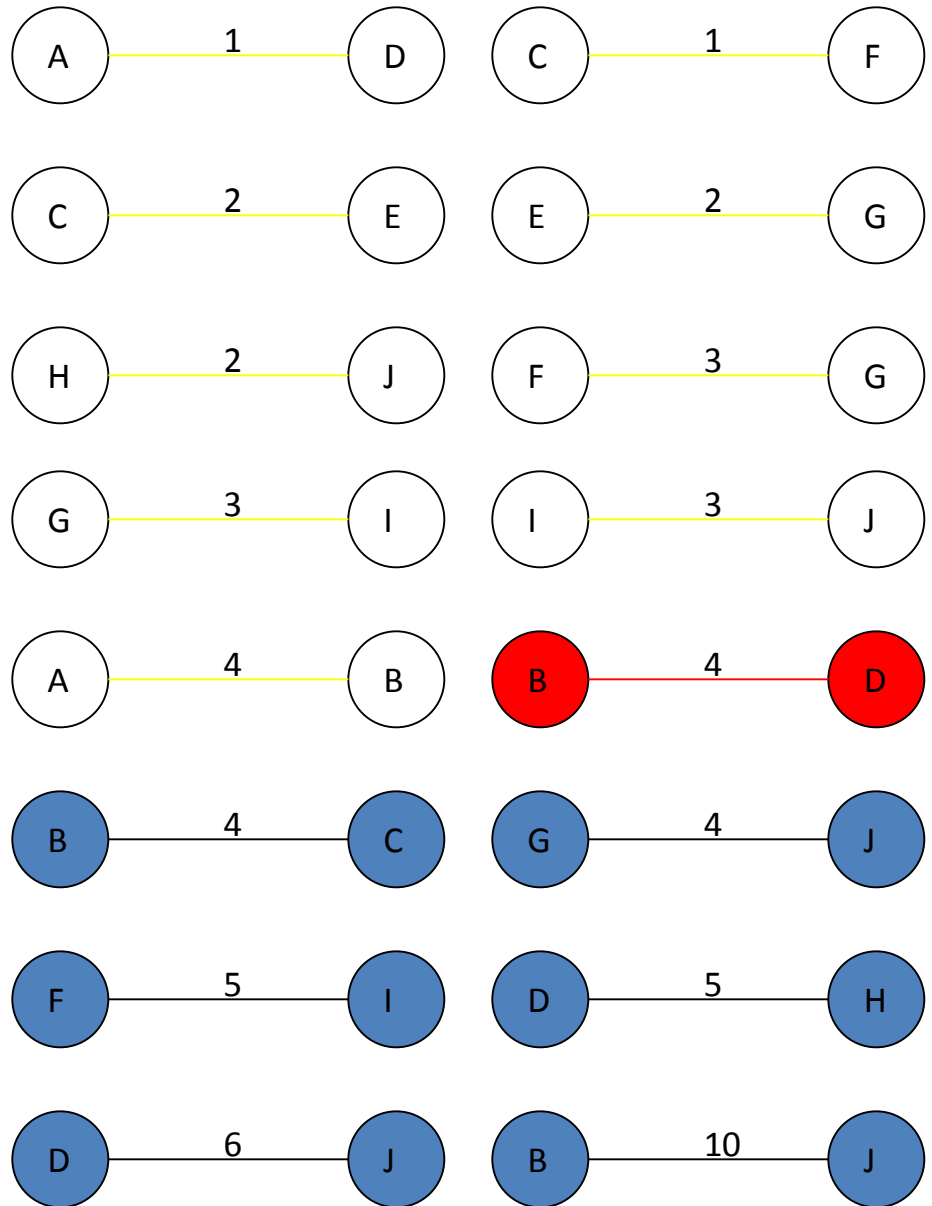
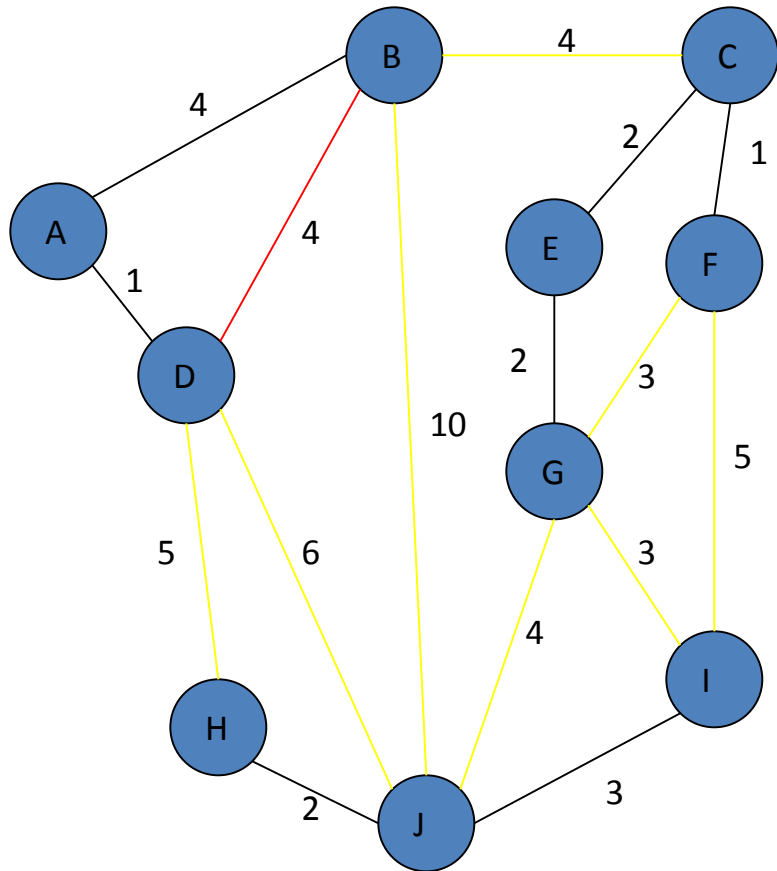


Add Edge



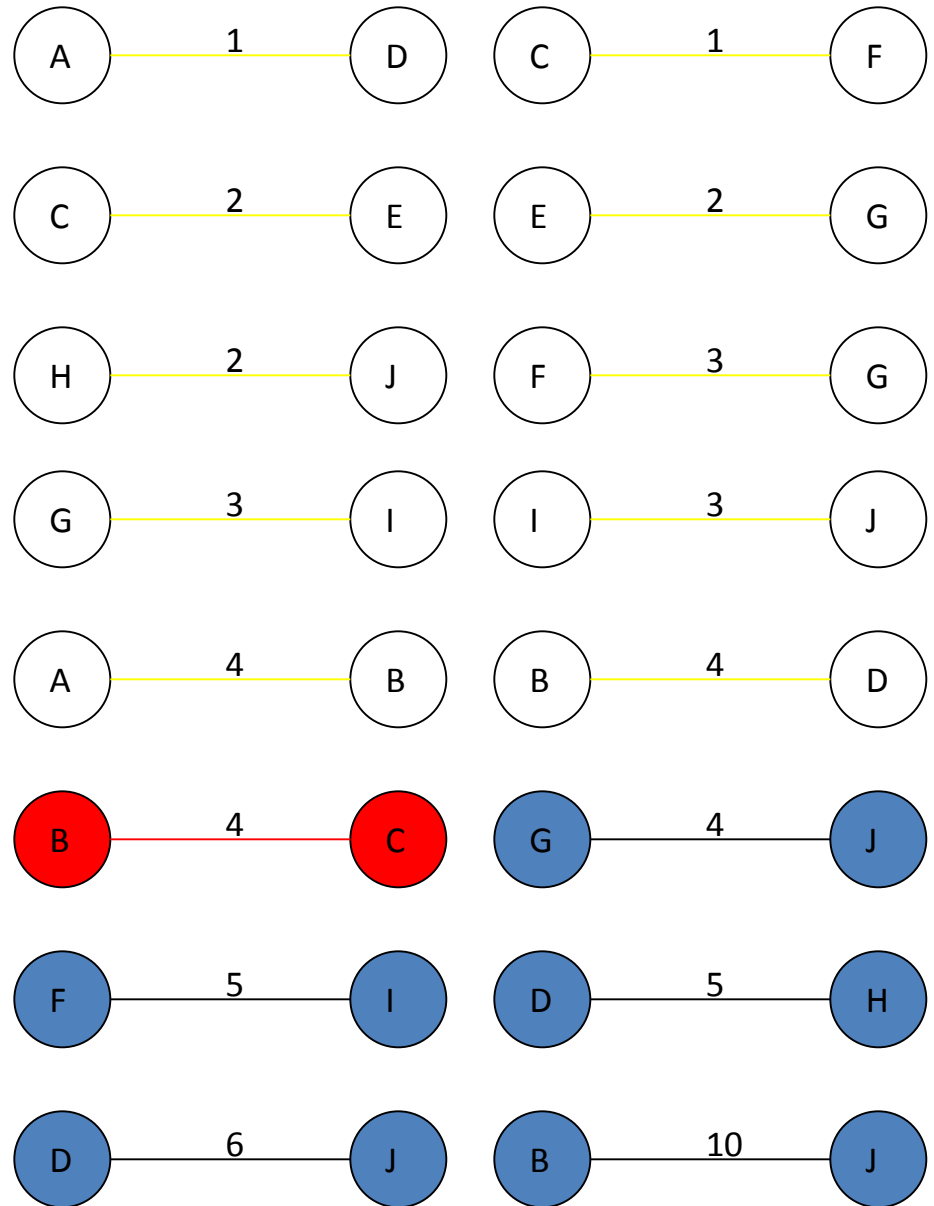
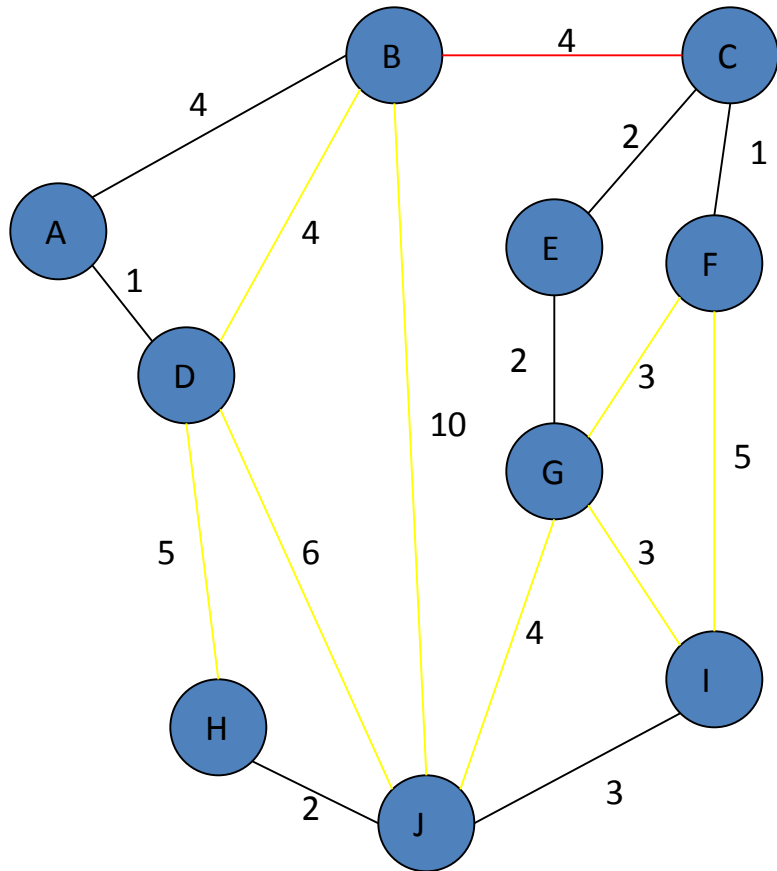
Cycle

Don't Add Edge

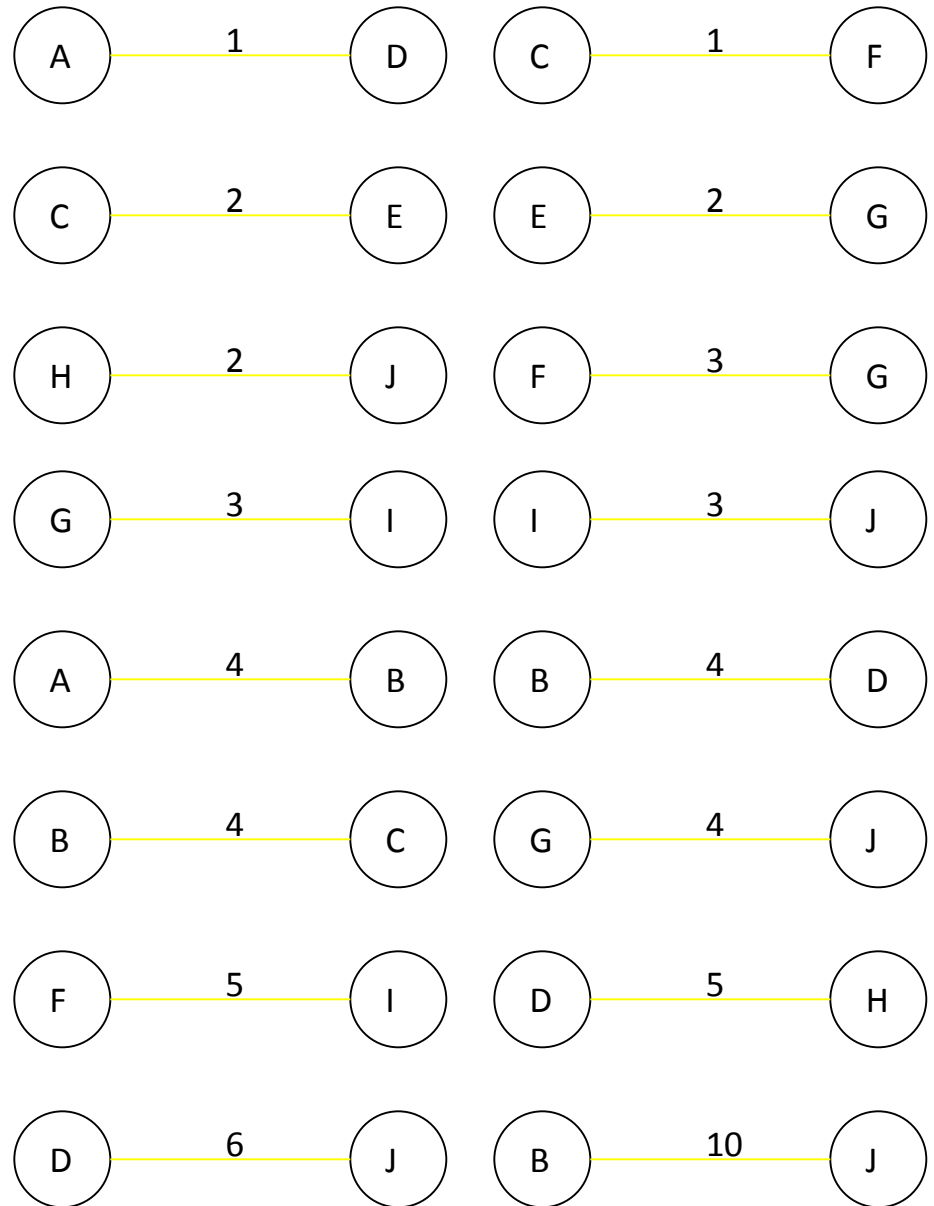
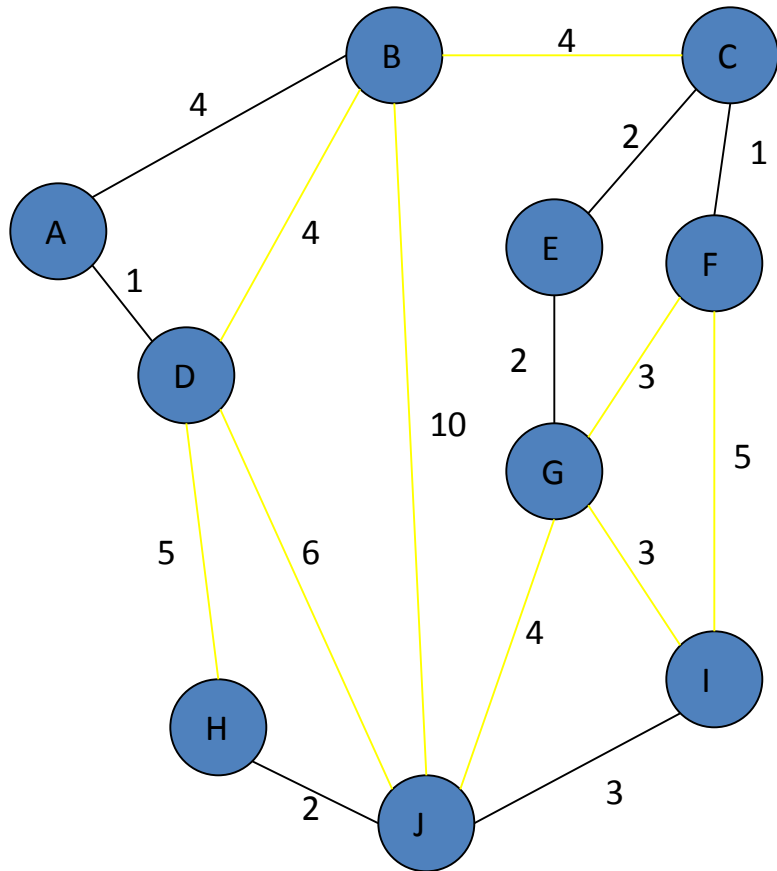


Higher than a dynamic threshold!

Don't Add Edge



Final -> 3 clusters



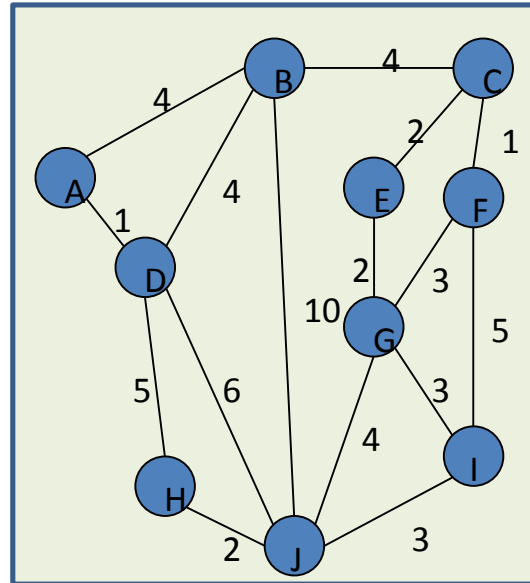
Example modified from Jonathan Davis's slides

# Conclusion - FH

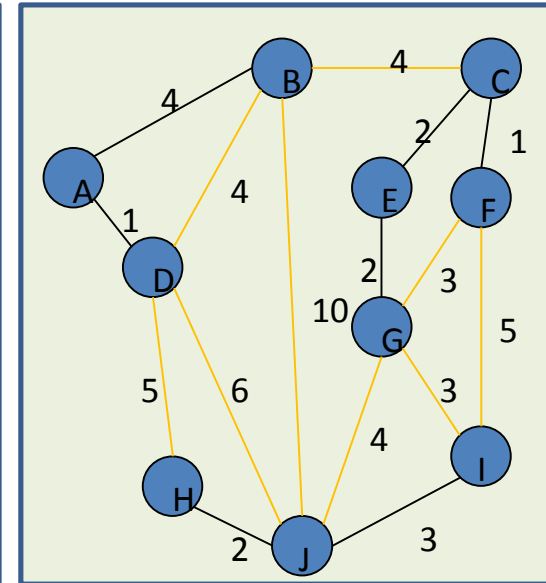
Efficient [graph-based](#) image segmentation, IJCV 2004



Image



Graph



Clustered result

|    | Performance                          | Efficiency     | Complexity<br>(# parameters) | Theoretical support |
|----|--------------------------------------|----------------|------------------------------|---------------------|
| FH | Reasonable<br>(good for superpixels) | <b>Fastest</b> | <b>Simple</b>                | Empirical           |

It is a good choice for general purpose applications if you don't mind the following drawbacks:

1. Sensitive to noise in images.
2. The shape of the segments might be strange.



# Methods

Ncut (Normalized Cut)

FH (Efficient graph-based image segmentation)

**Mean Shift**

gPb-OWT-UCM

ISCRA (Image Segmentation by Cascaded Region Agglomeration)

Citation: 6311  
(since year 2002)



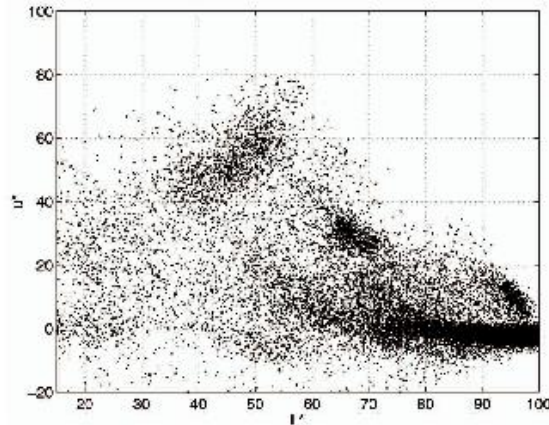
# Mean Shift

Mean shift: a robust approach toward feature space analysis, PAMI 2002

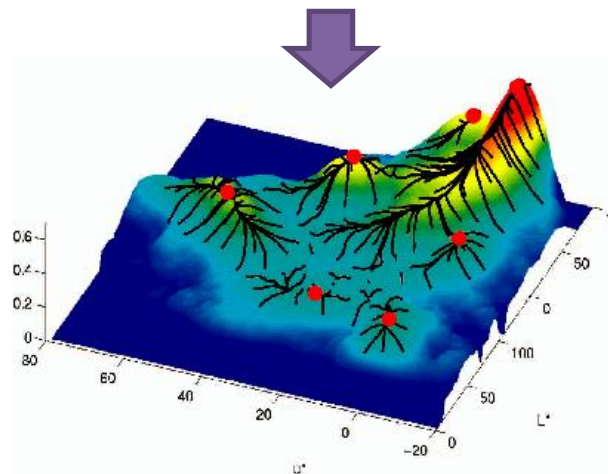


Image

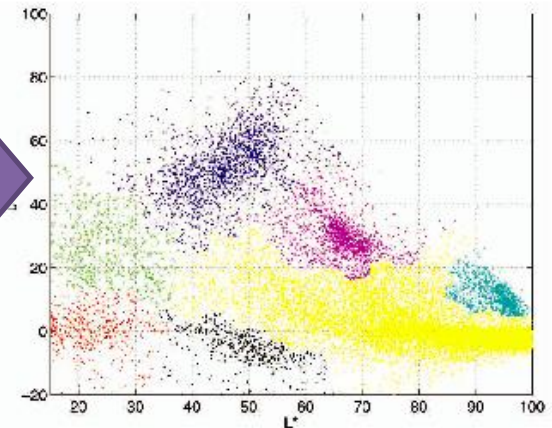
Projection on  $(L^*, u^*)$  space



In fact, mean shift usually work on 5 dimensions feature space (including 3 color channels and x, y for spatial location).



Mean shift (mode seeking)



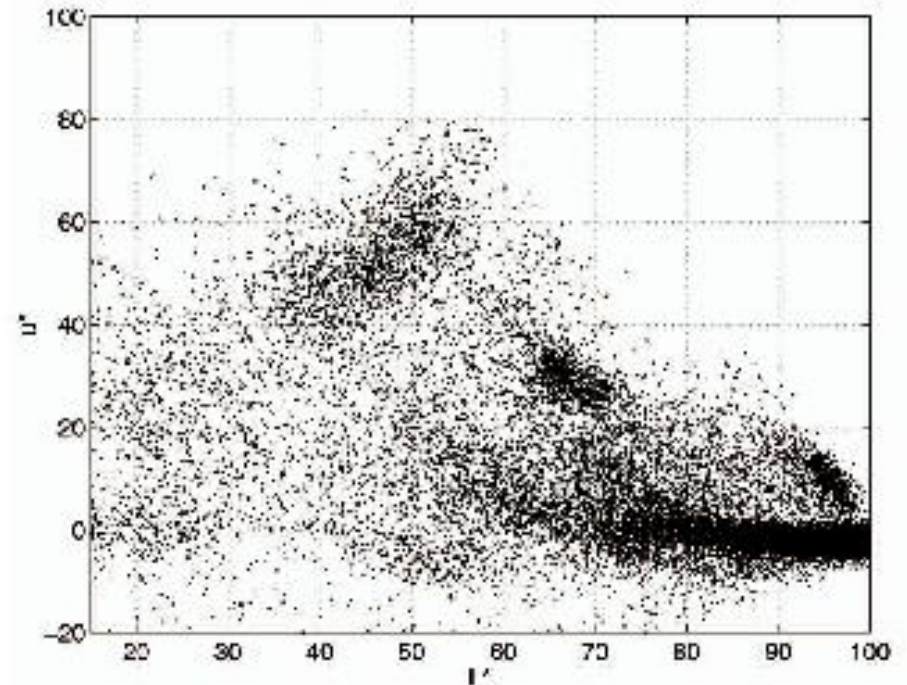
Clustered result

# Mean Shift Example

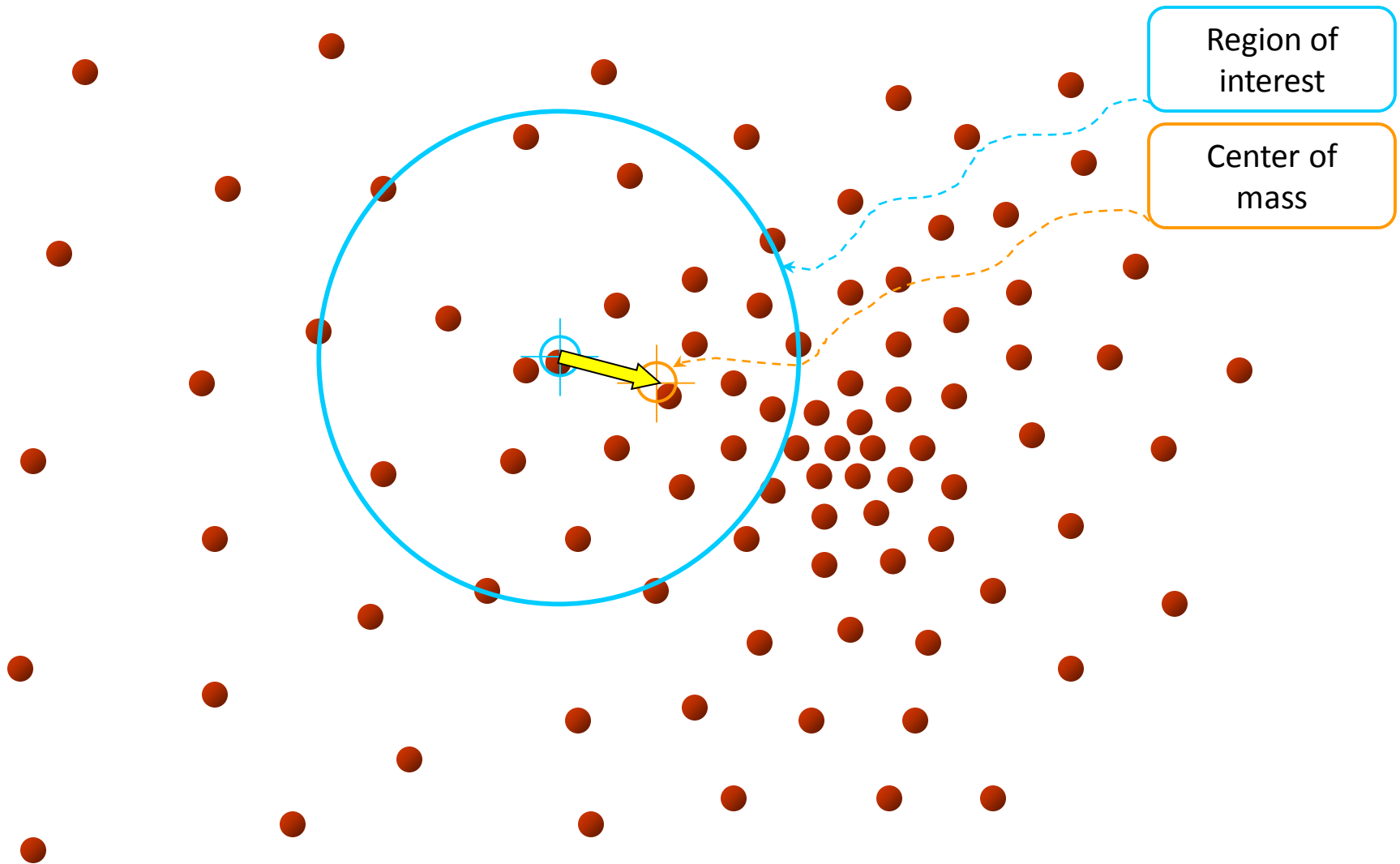
Projection on  $(L^*, u^*)$  space



Image

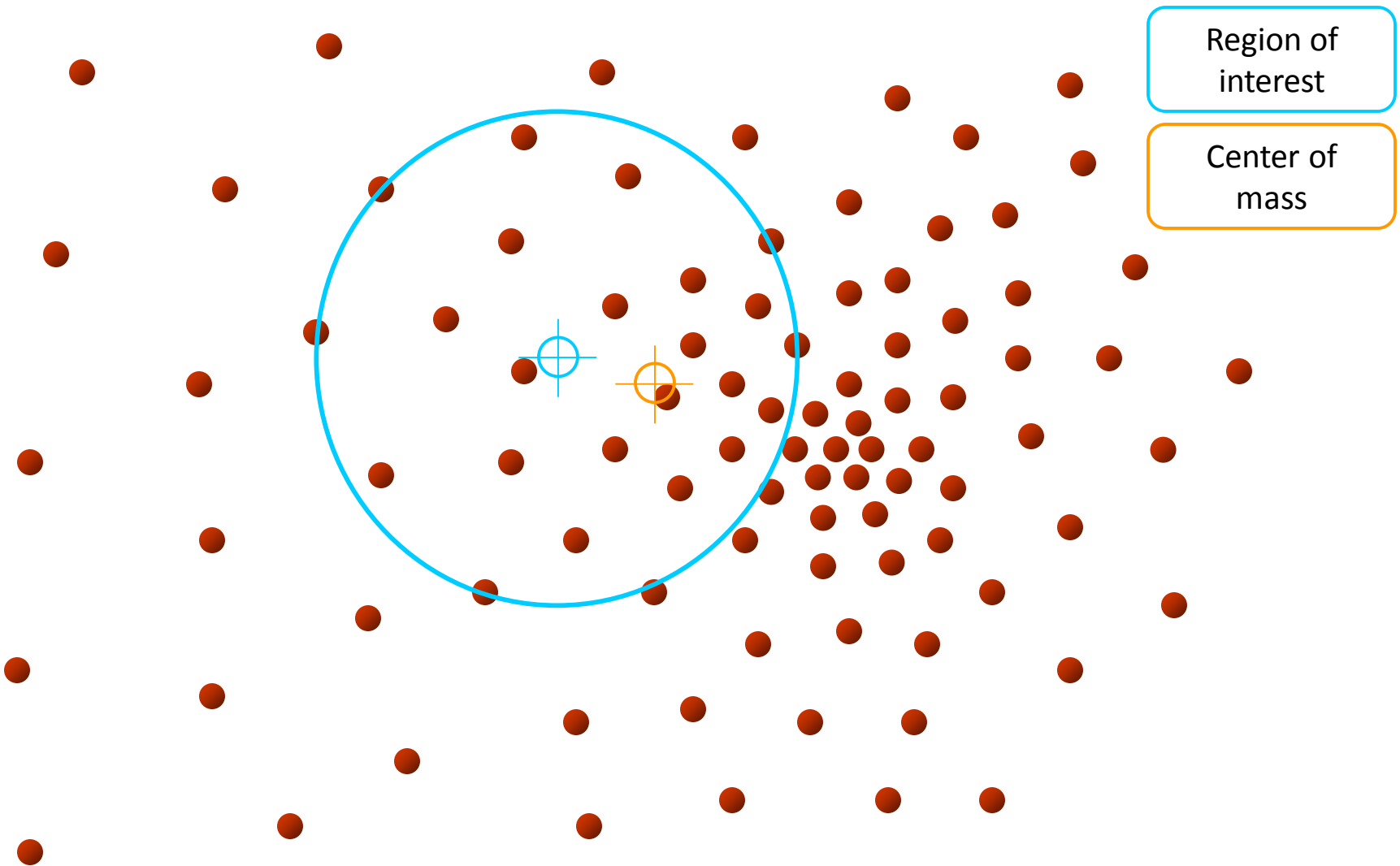


# Intuitive Description



Objective : Find the densest region

# Intuitive Description

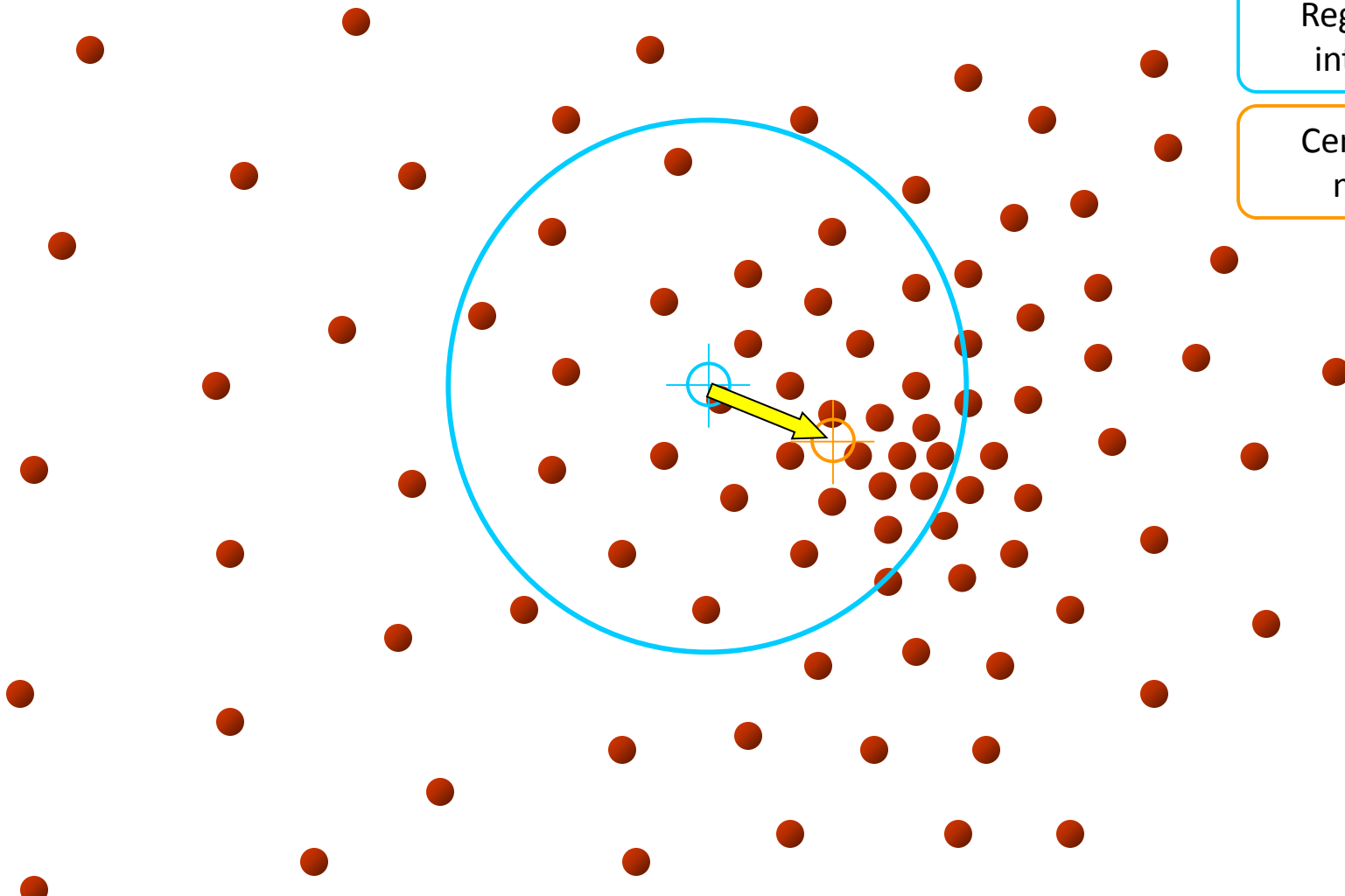


Objective : Find the densest region

# Intuitive Description

Region of  
interest

Center of  
mass

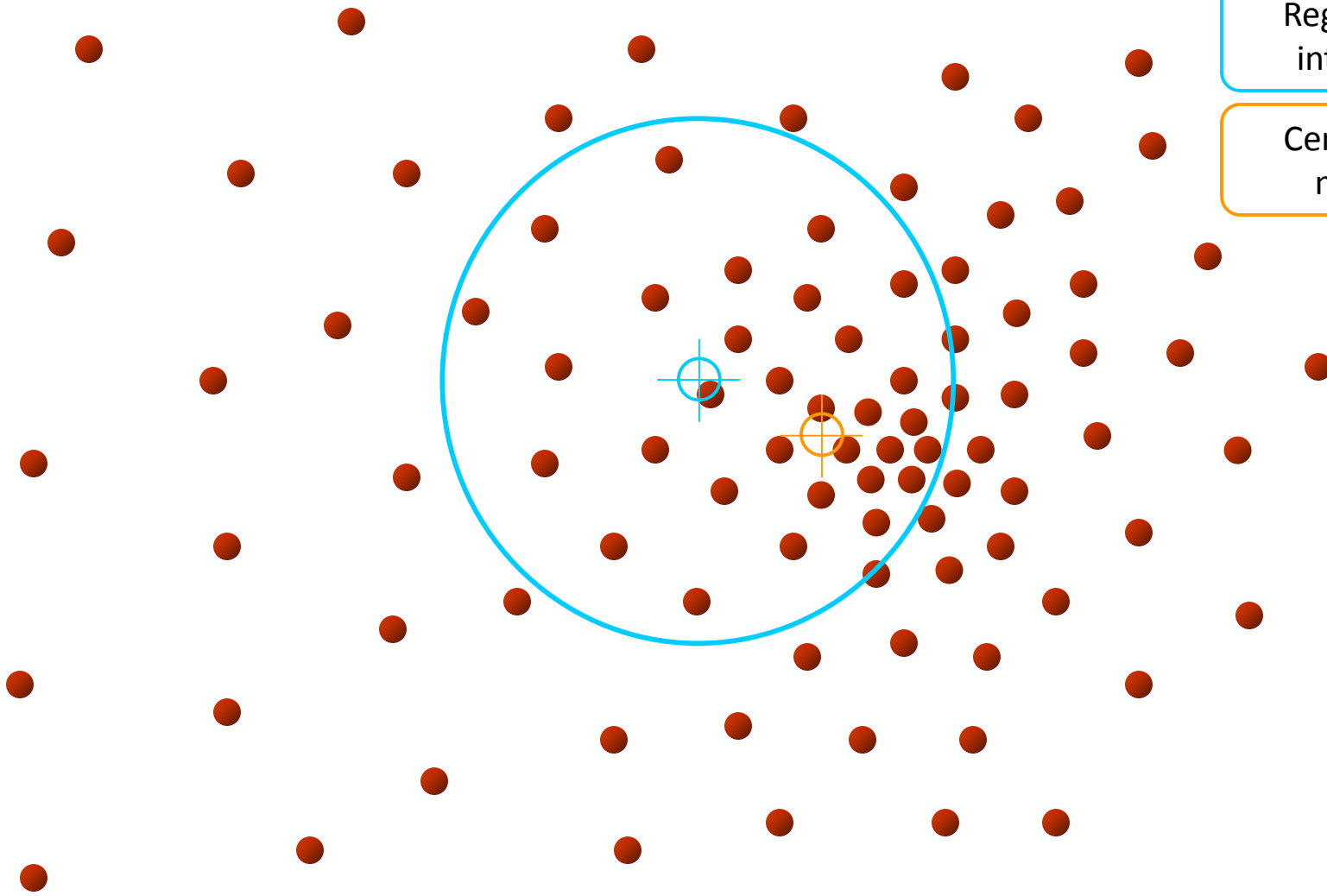


Objective : Find the densest region

# Intuitive Description

Region of  
interest

Center of  
mass

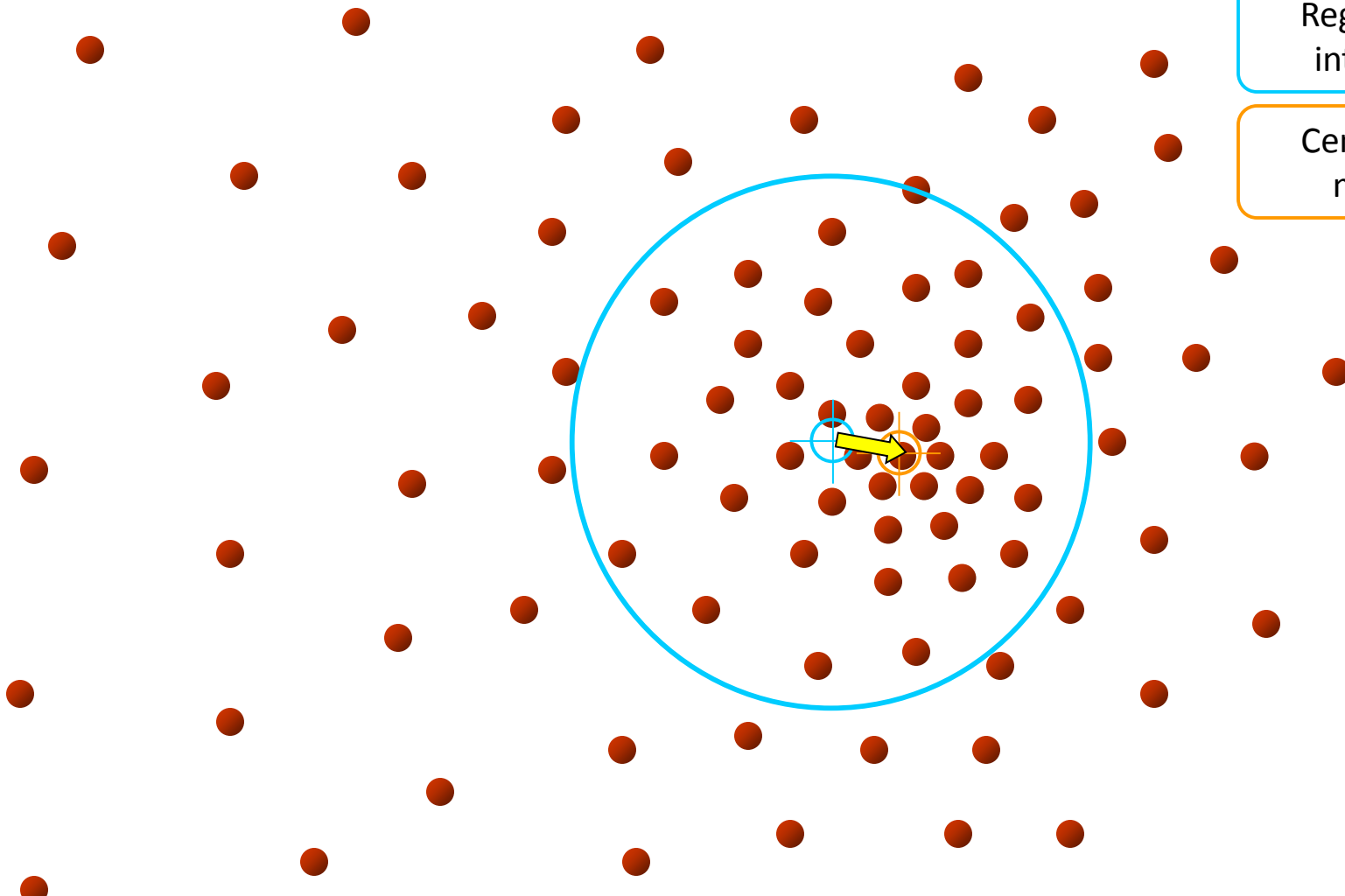


Objective : Find the densest region

# Intuitive Description

Region of  
interest

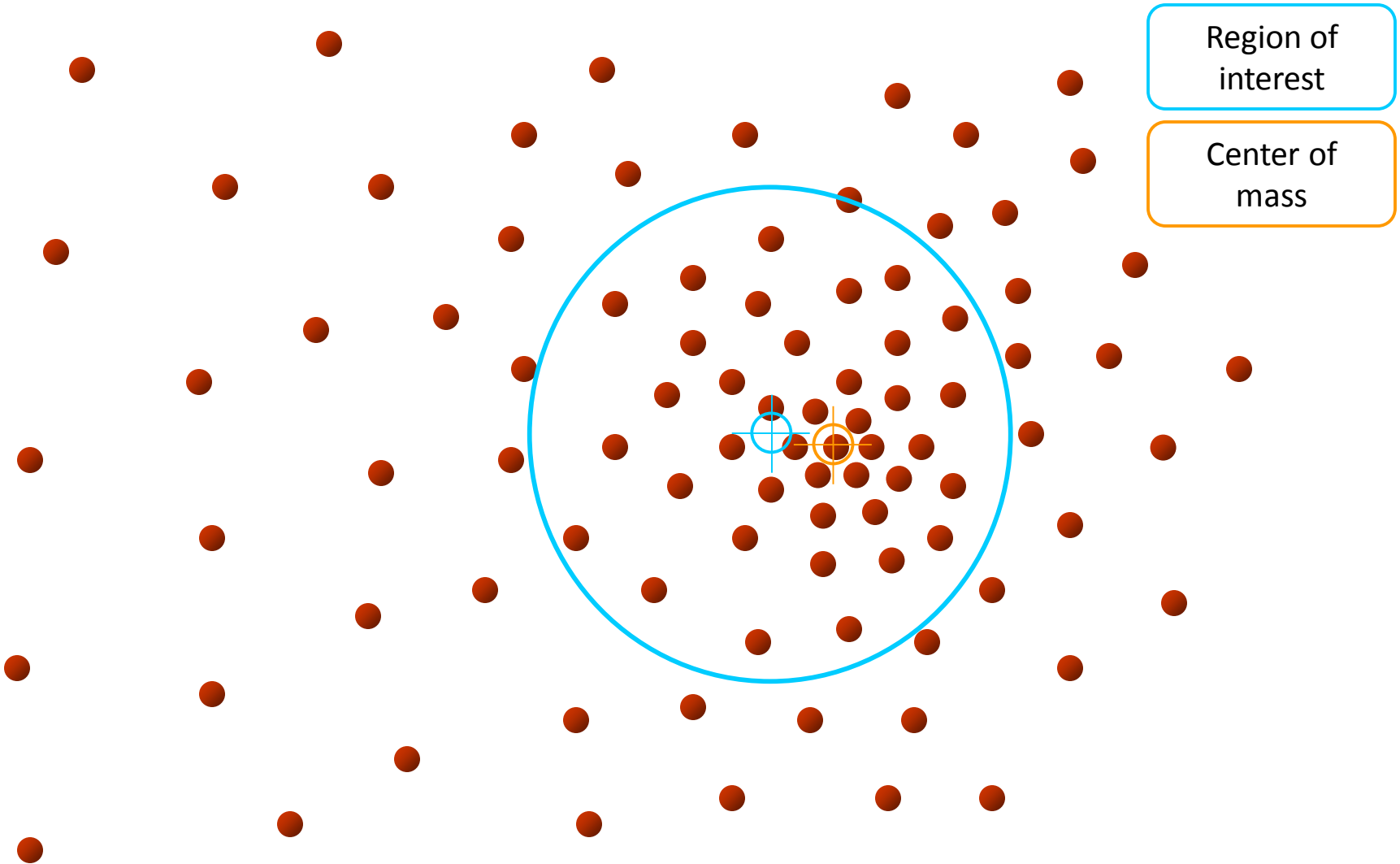
Center of  
mass



Objective : Find the densest region

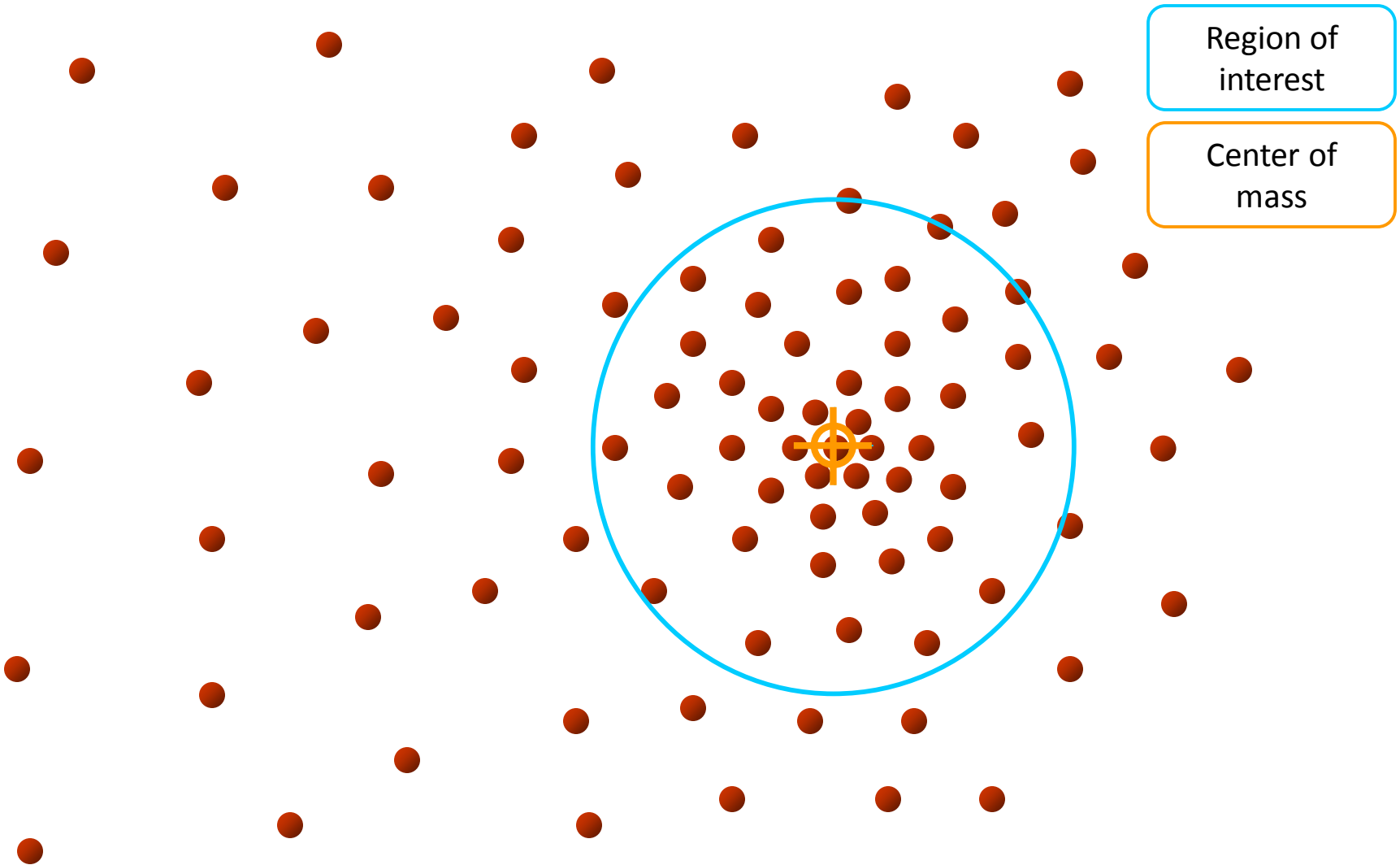


# Intuitive Description



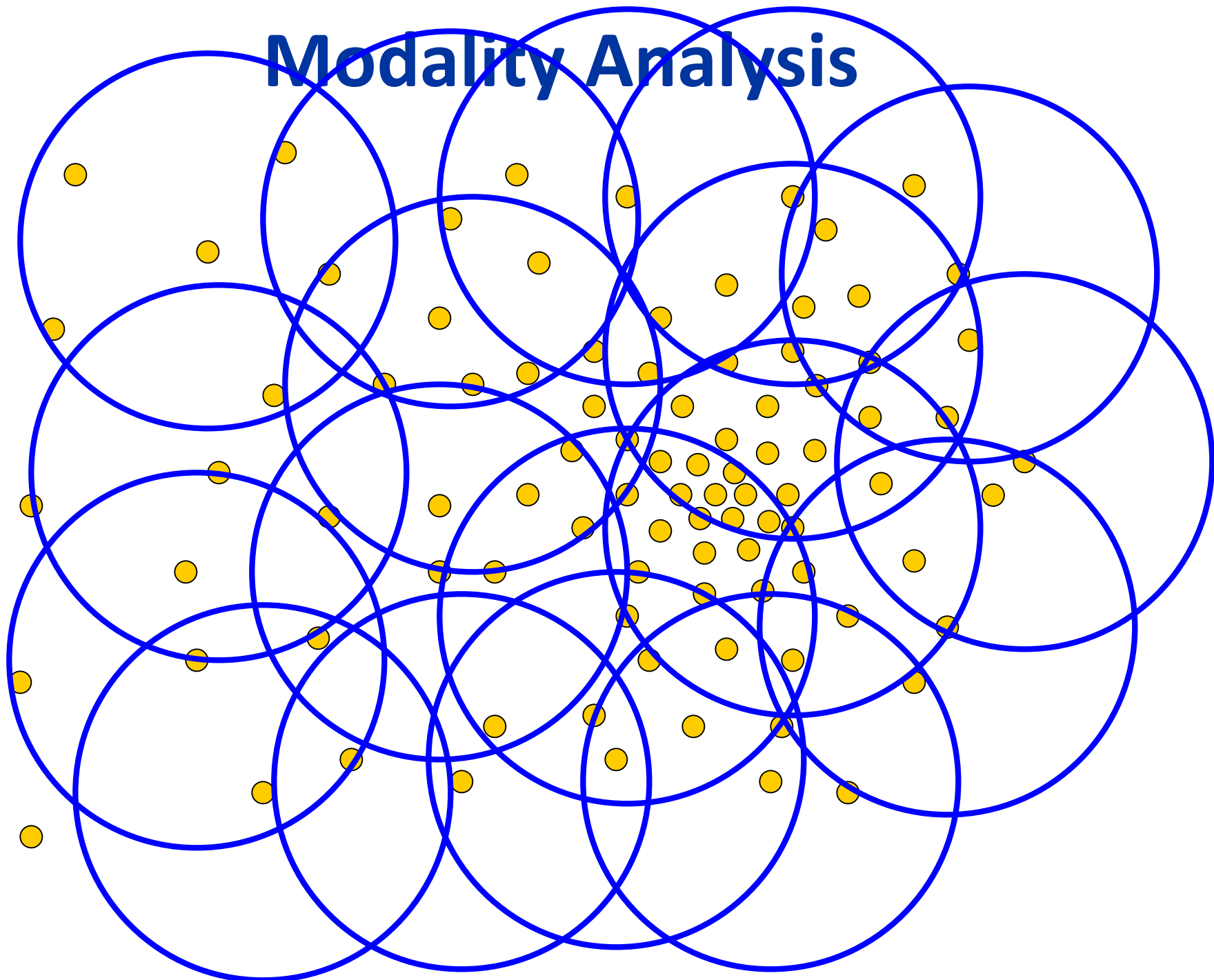
Objective : Find the densest region

# Intuitive Description

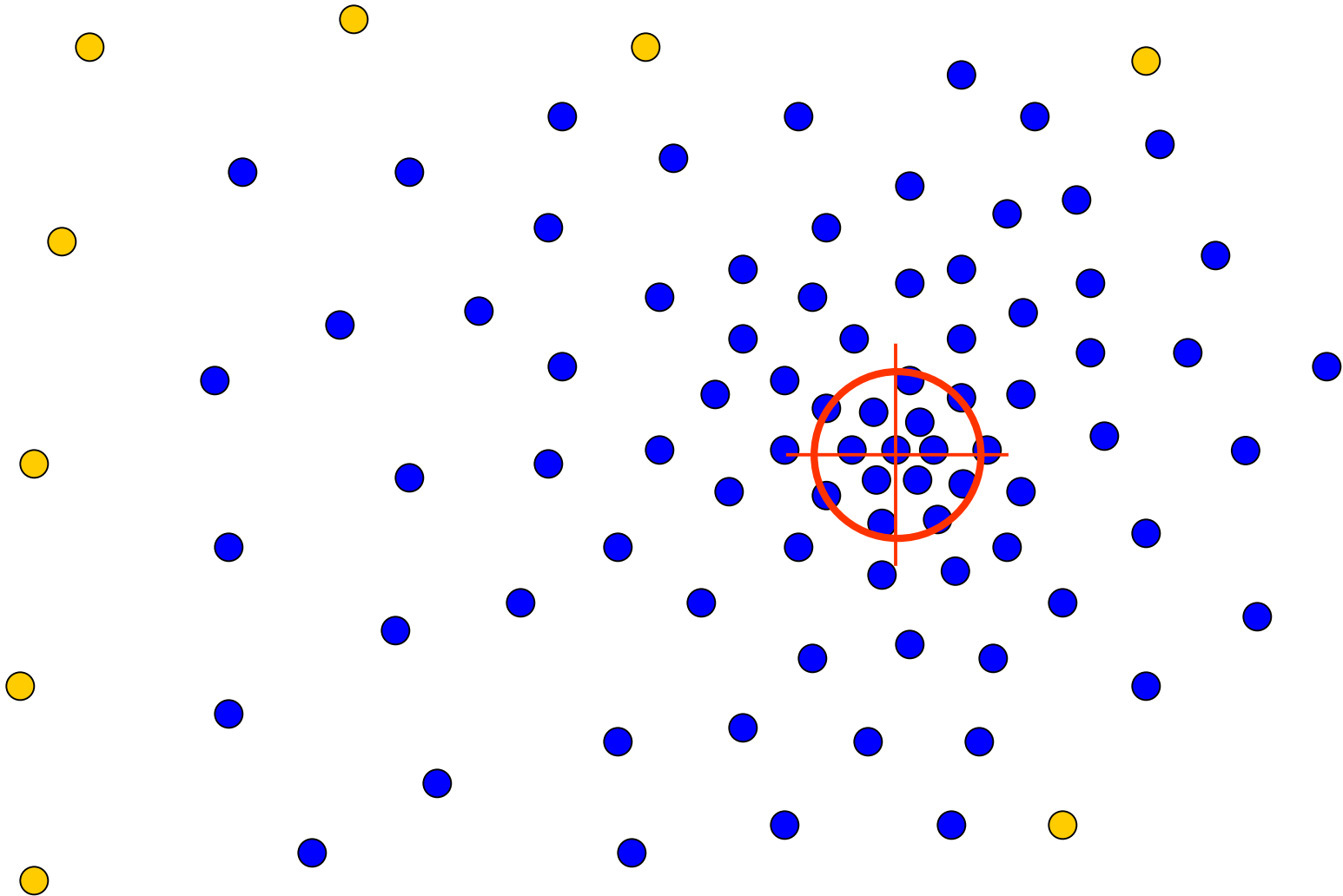


Objective : Find the densest region

# Modality Analysis



# Modality Analysis

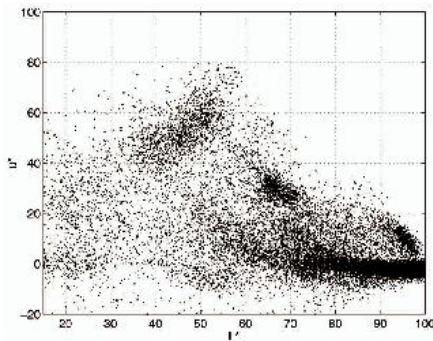


The blue data points were traversed by the windows towards the mode

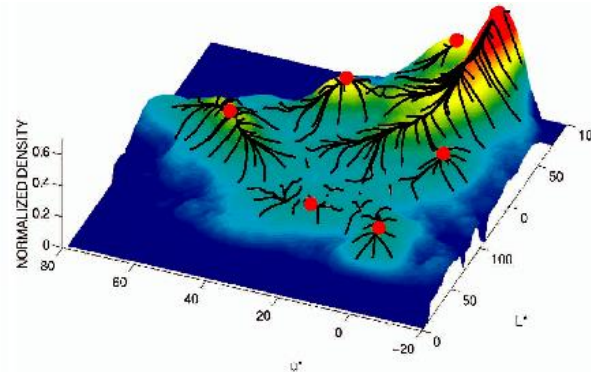
Example excerpted from Yaron Ukrainitz & Bernard Sarel's slides

# Conclusion - Mean Shift

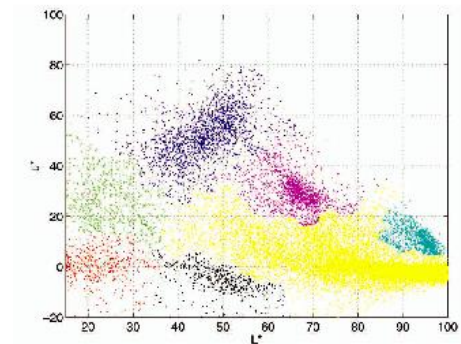
2D ( $L^*u$ ) space representation



Mean shift (mode seeking)



Final clusters

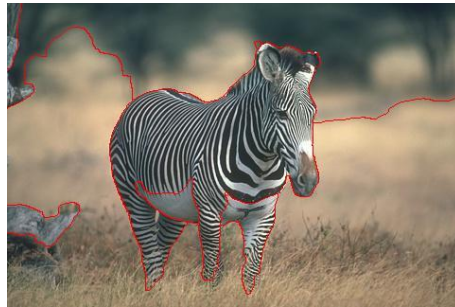


|            | Performance                             | Efficiency | Complexity<br>(# parameters) | Theoretical support |
|------------|---|------------|------------------------------|---------------------|
| Mean shift | Reasonable<br>(↑ for oversegmentations) | Reasonable | Simple                       | Good                |

It is a good choice for general purpose applications if you don't mind the following drawback:  
Slow in high resolution images.

# Common limitation of above methods

- Basic assumption:
  - color in different segments should be different



- How to integrate more features (e.g. texture)?
- Can we learn to distinct segments using training data?

# Methods

Ncut (Normalized Cut)

FH (Efficient graph-based image segmentation)

Mean Shift

**gPb-OWT-UCM**

ISCRA (Image Segmentation by Cascaded Region Agglomeration)

Citation: 408 + 245

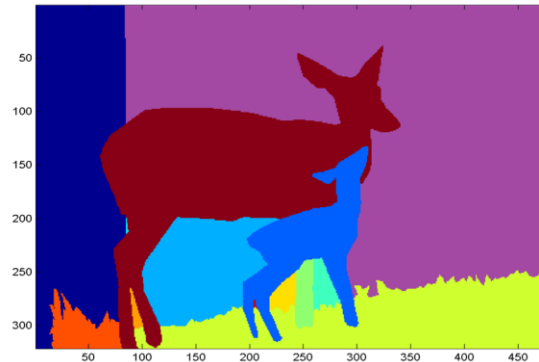
(since year 2011,2009)



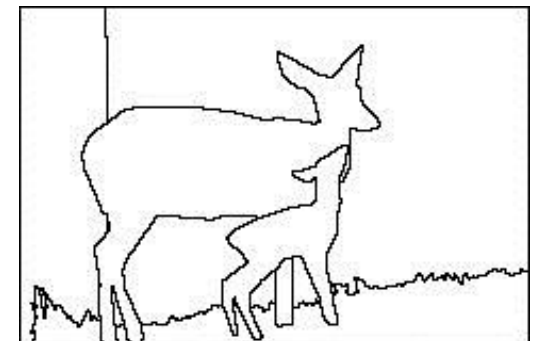
# gPb-OWT-UCM

Contour Detection and Hierarchical Image Segmentation, PAMI 2011

- Instead of clustering pixels, it solves the segmentation problem by **contour (object boundary) detection**.
  - **Classify** each pixel into “boundary” or “not boundary”
- It combines several different techniques
  - Supervised boundary classifier → NCut globalization → removing noise by watershed → minimal spanning tree merging



Clustering pixels  
search space  $\sim 13^{150000}$



Contour detection  
search space  $\sim 2^{150000}$



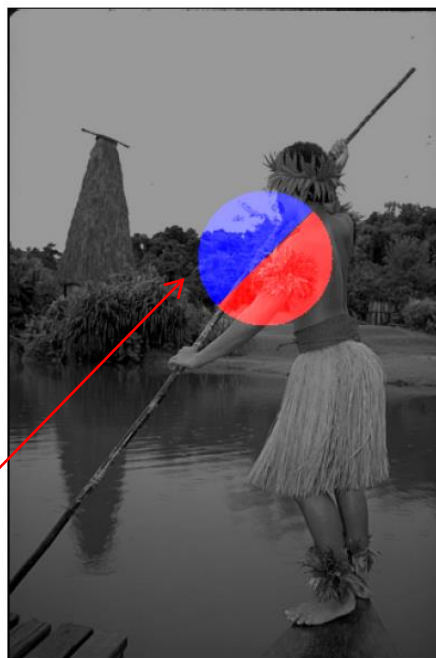
# mPb – Feature extraction for the classifier

- Feature: **Histogram difference** between two half disks
  - By assuming that the object boundaries are long enough straight lines

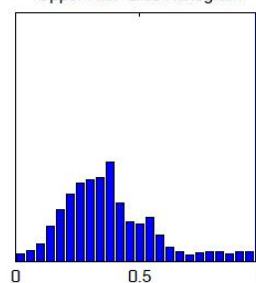
Input image



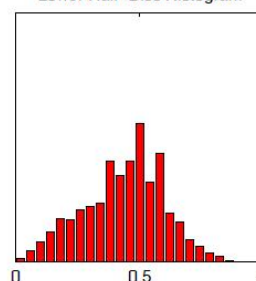
One feature channel



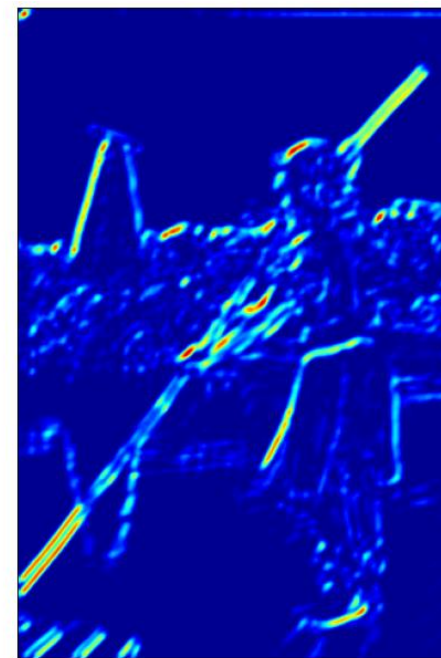
Upper Half-Disc Histogram



Lower Half-Disc Histogram



Feature on each pixel

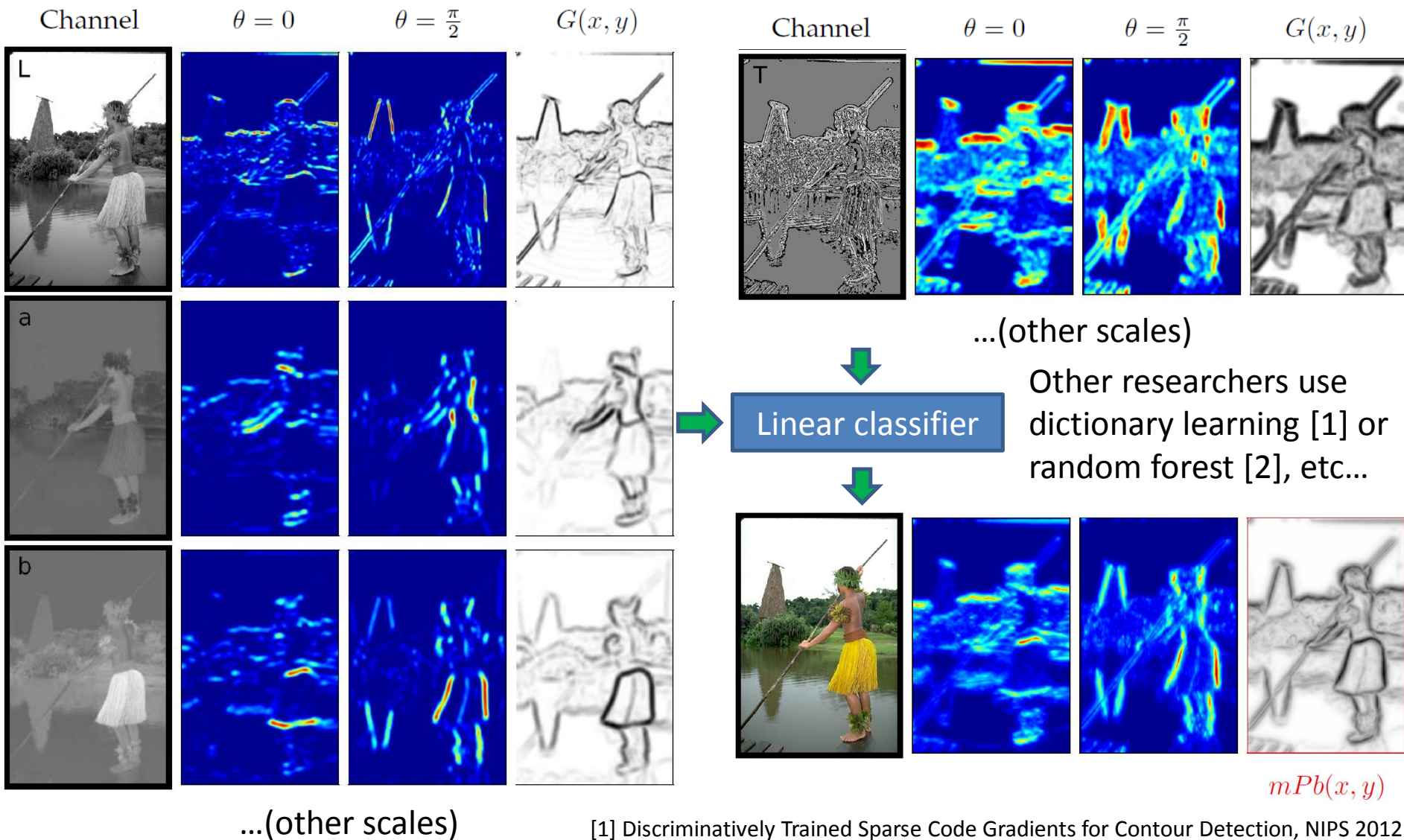


It will compute several different angles ( $\theta$ ) and disk sizes (scales).

Extract feature histograms within the disk

Compute the distance between histograms

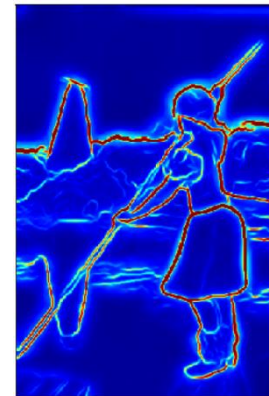
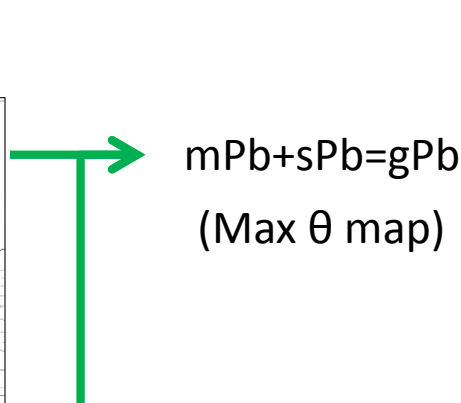
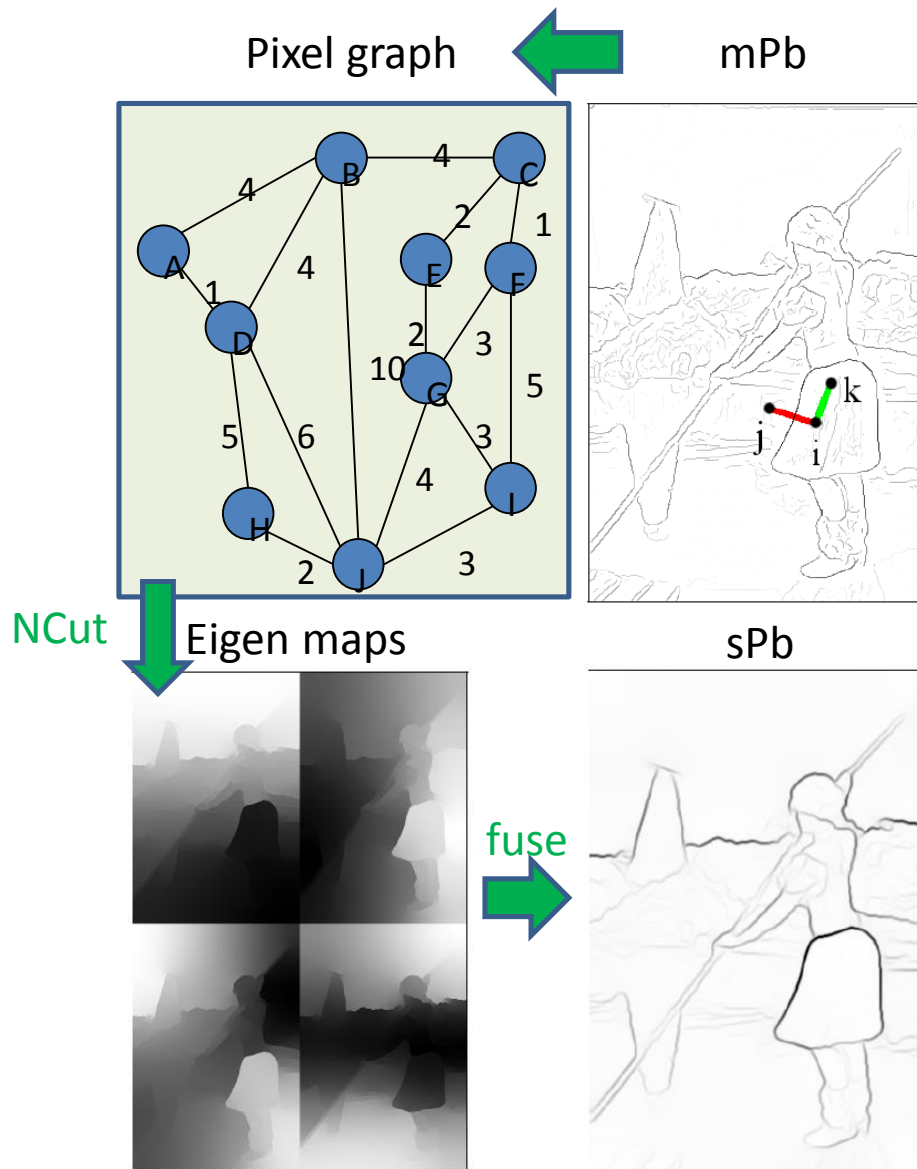
# mPb – Classification using multiple features



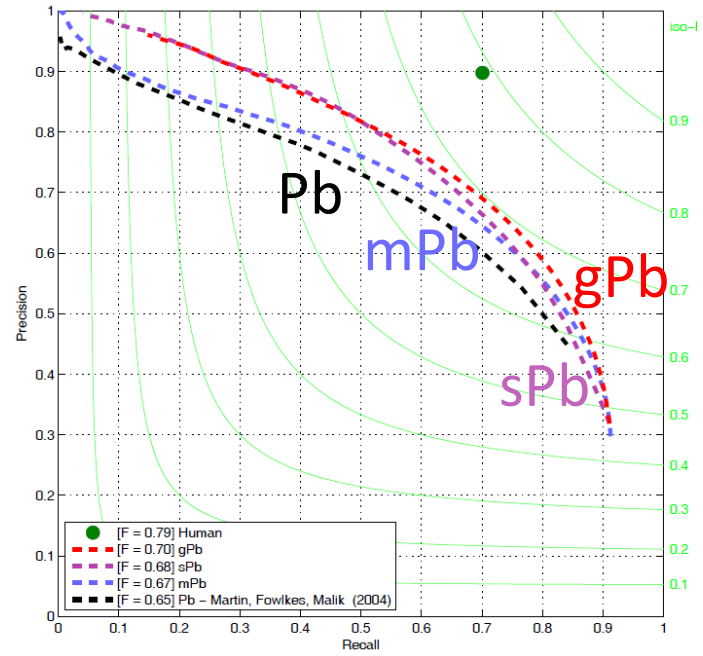
[1] Discriminatively Trained Sparse Code Gradients for Contour Detection, NIPS 2012

[2] Structure Forests for Fast Edge Detection, ICCV 2013

# gPb (global Probability boundary)



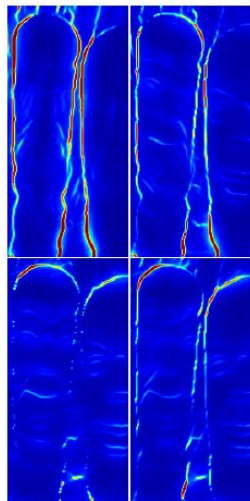
## Contour detection performance



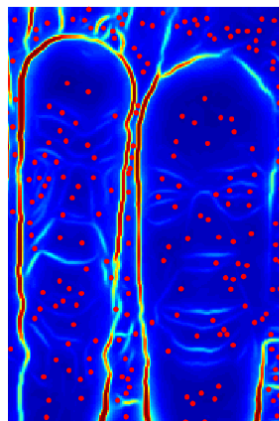
# OWT-UCM

input

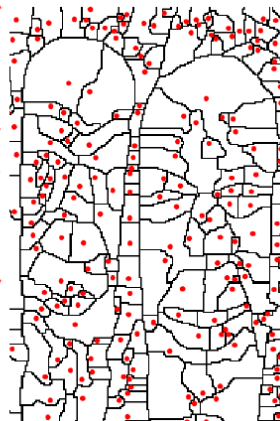
$gPb(\theta)$



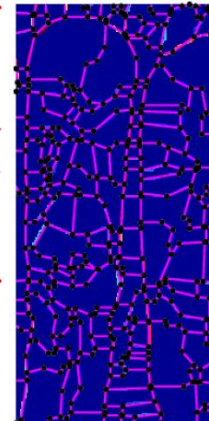
Oriented Watershed Transform (OWT)



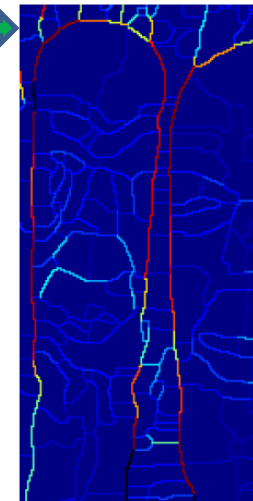
Local minimum



Watershed

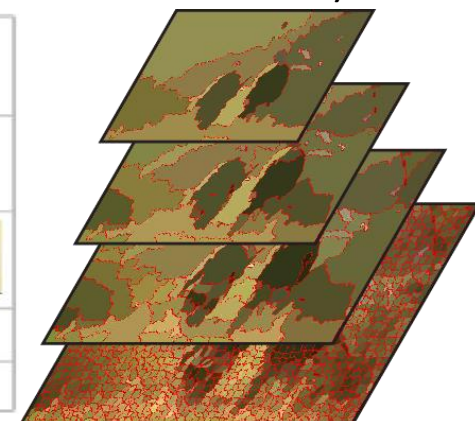
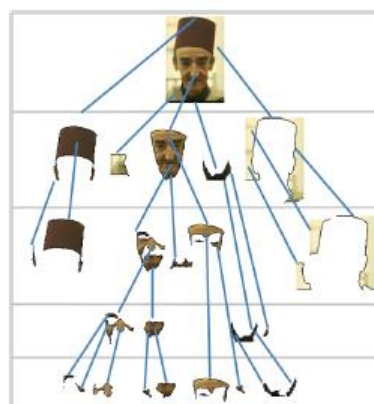
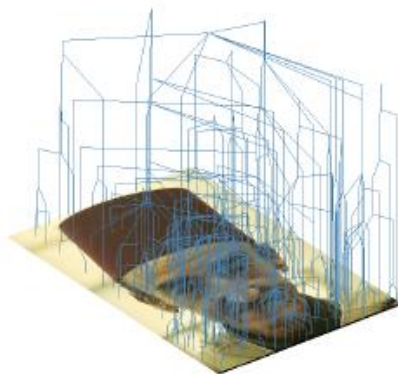
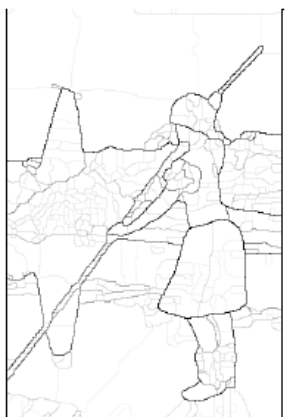


Remove noise



UCM (greedy merge like FH segmentation)  $\rightarrow$

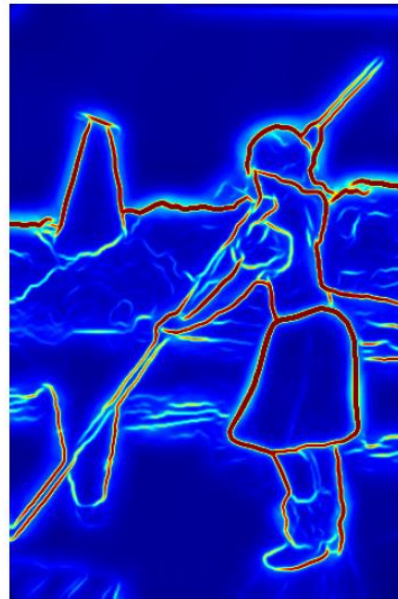
Hierarchical segmentation (output segmentations with # segments from 1 to  $\sim 1000$ )



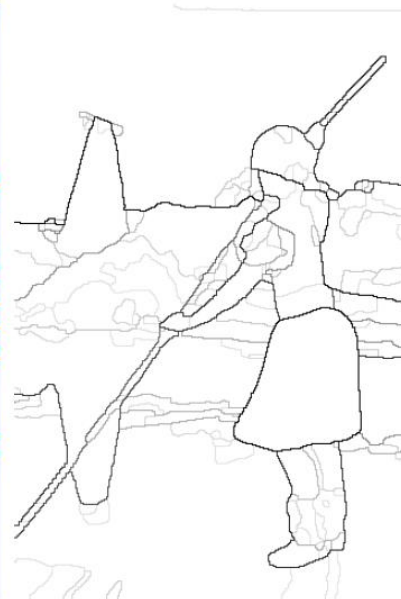
# Conclusion - gPb-OWT-UCM



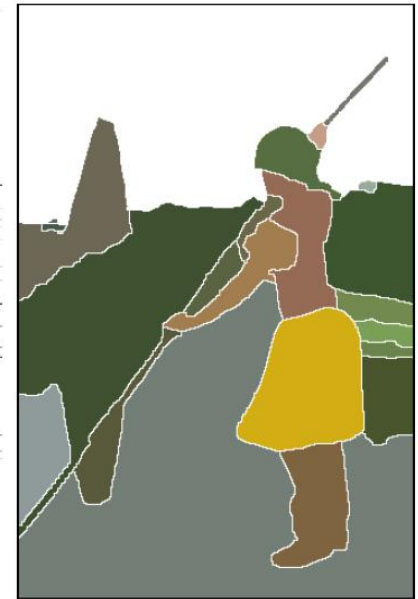
input



global Probability  
boundary (gPb)



Oriented watershed  
transform (OWT)



Ultrametric  
Contour Map (UCM)

|                                       | Performance | Efficiency | Complexity<br>(# parameters) | Theoretical<br>support |
|---------------------------------------|-------------|------------|------------------------------|------------------------|
| gPb-OWT-UCM<br>(need training) (2011) | Good        | Slow       | Complex                      | Empirical              |

It outperforms all methods which don't use gPb, and is widely used in recognition applications.

The concept of contour detection could be easily generalized to RGB-D images 61

# Methods

Ncut (Normalized Cut)

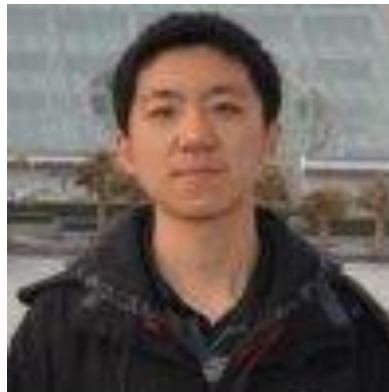
FH (Efficient graph-based image segmentation)

Mean Shift

gPb-OWT-UCM

**ISCRA (Image Segmentation by Cascaded Region Agglomeration)**

Citation: 1  
(since year 2013)



# Cascaded Region Agglomeration

Image Segmentation by Cascaded Region Agglomeration, CVPR 2013

gPb-  
OWT-  
UCM



input

Feature  
extraction

Classification

Greedy merging

Use the result of gPb-OWT-UCM

ISCRA



Feature  
extraction

Classification



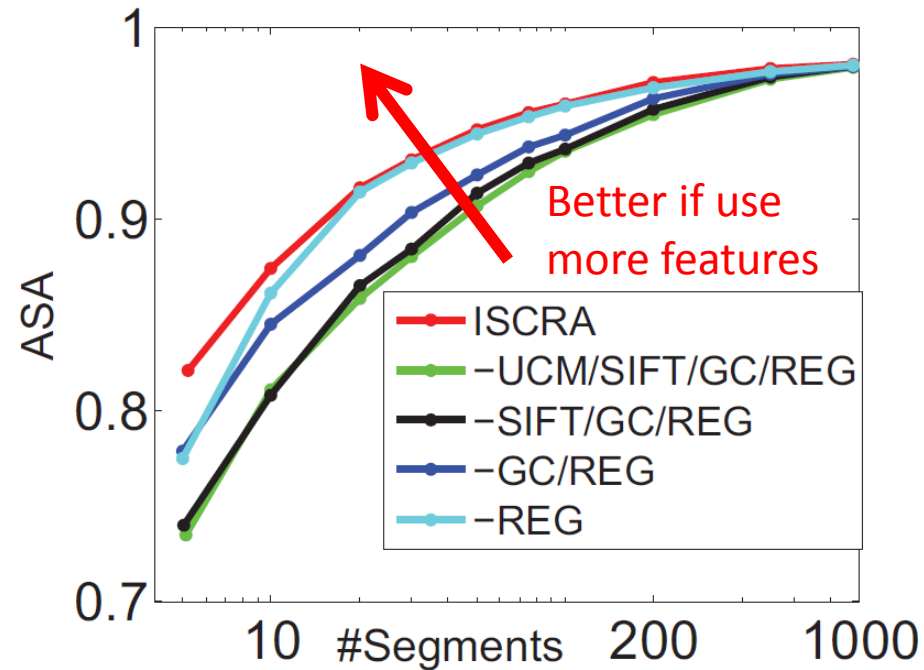
Feature  
extraction

Classification...



# Conclusion - ISCRA

- The performance is better than gPb-OWT-UCM because ISCRA
  - uses more features.
  - uses the different classifiers for different scales (layer of hierarchical).
    - could see more global features
  - **does not** assume that object boundaries are straight lines in the feature extraction.



|                                      | Performance                       | Efficiency                   | Complexity<br>(# parameters) | Theoretical support |
|--------------------------------------|-----------------------------------|------------------------------|------------------------------|---------------------|
| ISCRA (2013+)<br>(code not released) | <b>Best</b><br>(state-of-the-art) | Slowest<br>(need to use gPb) | Complex                      | Empirical           |



# Experimental Results

**Berkeley Segmentation Dataset (BSDS)**

**Evaluation - Quantitative Results**

**How much room left for improvement?**

# Berkeley Segmentation Dataset (BSDS)

- 500 natural images
- Image size:  $321 \times 481$  (or  $481 \times 321$ )
  - # ground truth segmentations per image  $\geq 4$ 
    - Manually segmented by different human subjects
    - The quality of ground truth is pretty high.
- Examples



Citation: 1884 (since year 2001)



Image

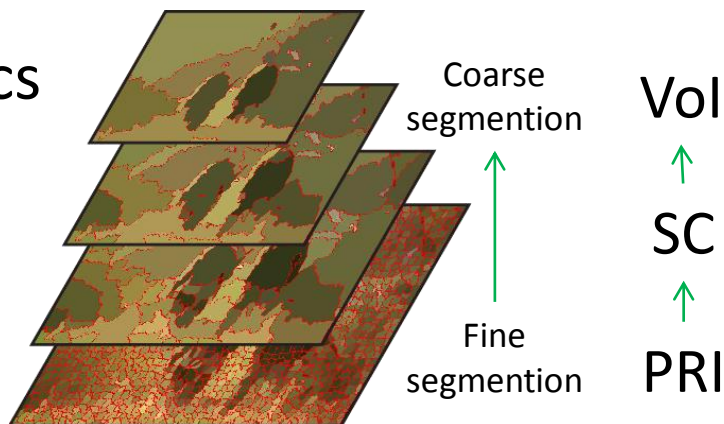
Ground truths

# Quantitative Evaluation Criteria

|             | BSDS 300 (100 test images) |             |             | BSDS 500 (200 test images) |             |             |
|-------------|----------------------------|-------------|-------------|----------------------------|-------------|-------------|
|             | SC (↑)                     | PRI (↑)     | Vol (↓)     | SC (↑)                     | PRI (↑)     | Vol (↓)     |
| NCut        | 0.44                       | 0.75        | 2.18        | 0.45                       | 0.78        | 1.89        |
| FH          | 0.51                       | 0.77        | 2.15        | 0.52                       | 0.80        | 2.21        |
| Mean shift  | 0.54                       | 0.78        | 1.83        | 0.54                       | 0.79        | 1.85        |
| gPb-OWT-UCM | 0.59                       | <b>0.81</b> | 1.65        | <b>0.59</b>                | <b>0.83</b> | 1.69        |
| ISCRA       | <b>0.60</b>                | <b>0.81</b> | <b>1.61</b> | <b>0.59</b>                | 0.82        | <b>1.60</b> |

Outperforms gPb-OWT-UCM more significantly on other datasets such as [MSRC](#) or [SBD](#)

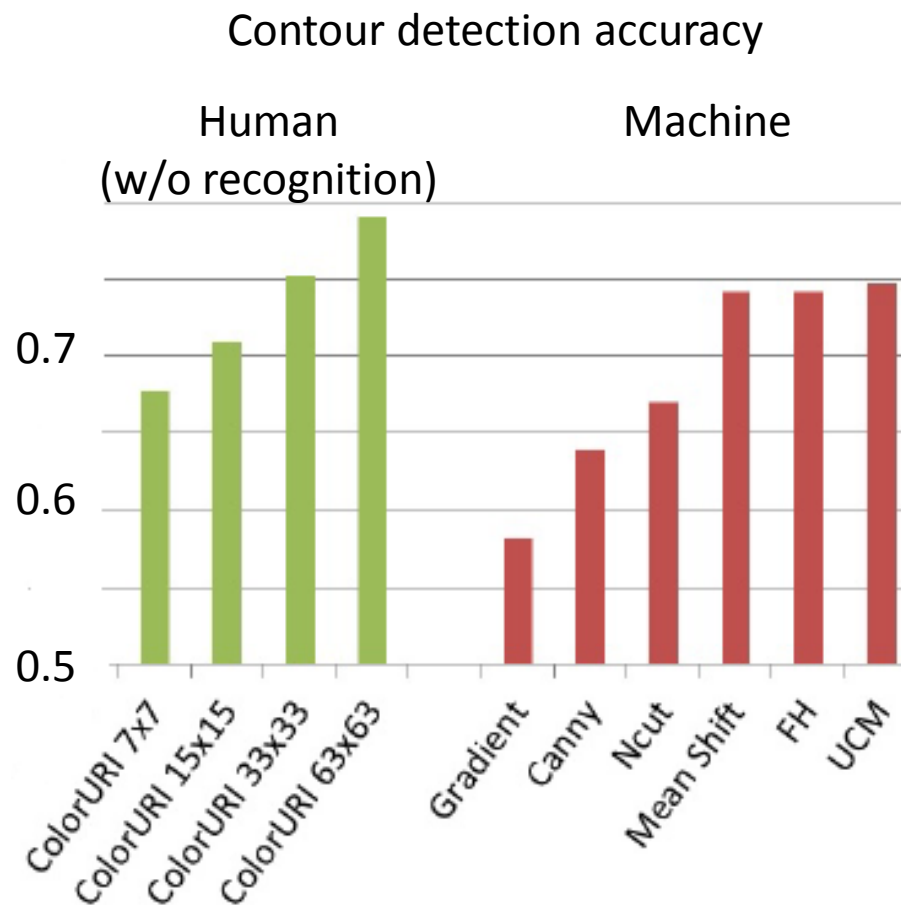
## ■ Criteria characteristics



([details](#))

# How much room left for improvement?

- SUN dataset
  - Subset: 185 images
- Interesting facts
  - Learning based method might overfit training data
    - gPb-OWT-UCM is not significantly better
  - Local information has been exhausted
  - There is not much room for improvement



The Role of Image Understanding in Contour Detection, CVPR2012

# Conclusion

- After years of research efforts, researchers find that totally unsupervised segmentation is an ill-posed problem, so the methods or problems required learning are actually more well-defined and practical
  - I believe that it is **very hard** to improve performance in totally unsupervised segmentation problem without overfitting datasets
  - I believe that it is **easier and more meaningful** to improve performances in supervised setting or other desired characteristics such as preventing overfitting, efficiency of training and testing, complexity and theoretical support ...

|                                       | Performance                        | Efficiency                   | Complexity<br>(# parameters) | Theoretical support |
|---------------------------------------|------------------------------------|------------------------------|------------------------------|---------------------|
| NCut (2000)                           | Bad<br>(original version)          | Reasonable                   | Simple                       | Best                |
| FH (2004)                             | Reasonable<br>(↑ for superpixels)  | Fastest                      | Simple                       | Empirical           |
| Mean shift (2002)                     | Reasonable<br>(↑ for oversegments) | Reasonable                   | Simple                       | Good                |
| gPb-OWT-UCM<br>(need training) (2011) | Good                               | Slow                         | Complex                      | Empirical           |
| ISCRA (2013+)<br>(code not released)  | Best<br>(state of the art)         | Slowest<br>(need to use gPb) | Complex                      | Empirical           |

# Q & A

# References

- J. Shi and J. Malik, “Normalized cuts and image segmentation,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2000.
- D. Martin, C. Fowlkes, D. Tal, and J. Malik, “A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics,” IEEE International Conference on Computer Vision, 2001.
- D. Comaniciu, and P. Meer, “Mean shift: a robust approach toward feature space analysis,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, May 2002.
- P. Felzenszwalb, D. Huttenlocher, “Efficient graph-based image segmentation,” *International Journal of Computer Vision*, 2004.
- P. Arbeláez, M. Maire, C. Fowlkes, and J. Malik, “Contour Detection and Hierarchical Image Segmentation,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2011.
- Z Ren, G. Shakhnarovich, “Image Segmentation by Cascaded Region Agglomeration,” *IEEE Computer Vision and Pattern Recognition*, 2013.

# Appendix – NCut derivation

$$\text{Ncut}(A, B) := \text{cut}(A, B) \left( \frac{1}{\text{vol}(A)} + \frac{1}{\text{vol}(B)} \right)$$

$$\text{Let } \mathbf{f} = [f_1 \ f_2 \ \dots \ f_n]^T \text{ with } f_i = \begin{cases} \frac{1}{\text{vol}(A)} & \text{if } i \in A \\ -\frac{1}{\text{vol}(B)} & \text{if } i \in B \end{cases}$$

$$\mathbf{f}^T \mathbf{L} \mathbf{f} = \sum_{ij} w_{ij} (f_i - f_j)^2 = \sum_{i \in A, j \in B} w_{ij} \left( \frac{1}{\text{vol}(A)} + \frac{1}{\text{vol}(B)} \right)^2$$

(L=D-W)

$$\mathbf{f}^T \mathbf{D} \mathbf{f} = \sum_j d_j f_j^2 = \sum_{i \in A} \frac{d_i}{\text{vol}(A)^2} + \sum_{j \in B} \frac{d_j}{\text{vol}(B)^2} = \frac{1}{\text{vol}(A)} + \frac{1}{\text{vol}(B)}$$

$$\text{Ncut}(A, B) = \frac{\mathbf{f}^T \mathbf{L} \mathbf{f}}{\mathbf{f}^T \mathbf{D} \mathbf{f}} \rightarrow \boxed{\mathbf{L} \mathbf{f} = \lambda \mathbf{D} \mathbf{f}}$$



# Appendix – Quantitative Evaluation Criteria

- **Probabilistic Rand Index (PRI)  $\uparrow [0, 1]$** 
  - For any two pixels, what is the **probability** of the two pixels belonging to the same cluster (or different clusters) in the ground-truth set?
- **Variation of Information (Vol)  $\downarrow [0, \infty)$** 
  - How many **bits** are required to describe the **difference** between a test result and a ground truth?
- **Segmentation Covering (SC)  $\uparrow [0, 1]$** 
  - Suppose each segment in a ground truth is to be detected, what is the overall **quality of detections** in a test result according to IOU-overlap?
- **Achievable segmentation accuracy (ASA)  $\uparrow [0, 1]$** 
  - ASA is a **superpixel** performance upper bound measure. It gives the highest accuracy achievable for object segmentation that utilizes superpixels as units.

| Stress on \ Criteria | SC ( $\uparrow$ ) | PRI ( $\uparrow$ ) | Vol ( $\downarrow$ ) |
|----------------------|-------------------|--------------------|----------------------|
| Hierarchical level   | Middle            | Low                | High                 |
| Scale of objects     | Middle            | Small              | Large                |
| # segments           | Middle            | High               | Low                  |